Tunnelling and Underground Space Technology 70 (2017) 89–104

Contents lists available at ScienceDirect

Tunnelling and Underground Space Technology

journal homepage: www.elsevier.com/locate/tust

Probabilistic analysis of tunnels: A hybrid polynomial correlated function expansion based approach

Dipaloke Majumder[a,1], Souvik Chakraborty[b,1,*], Rajib Chowdhury[a]

[a] Department of Civil Engineering, Indian Institute of Technology Roorkee, Roorkee 247667, India
[b] Department of Aerospace and Mechanical Engineering, University of Notre Dame, Notre Dame 46556, USA

ARTICLE INFO

Keywords:
Rock tunnels
Uncertainty
H-PCFE
Covariance function

ABSTRACT

This paper presents a novel approach for the analysis of tunnels in the presence of uncertainties. The proposed approach, referred to here as hybrid polynomial correlated function expansion (H-PCFE), performs a bi-level approximation: first on global scale via polynomial correlated function expansion (PCFE) and second on local scale via Kriging. While PCFE approximates the overall trend of the output response by using extended bases, Kriging utilizes covariance function to track the local variations. Additionally, a novel homotopy algorithm is utilized for estimating the unknown coefficients associated with the bases. The proposed approach has been utilized for analysis of two benchmark tunnel problems. In order to demonstrate the superior performance of the proposed approach, results obtained have been compared with those obtained using radial basis function (RBF) and Kriging. For both the problems, the proposed H-PCFE based approach yields highly accurate results outperforming both RBF and Kriging. Additionally, the proposed approach is computationally efficient as indicated by the convergence plots that illustrate the rapid decrease in prediction error with the increase in number of training points.

1. Introduction

In view of increasing population and scarcity of space on ground surface, the trend of utilizing underground space is growing rapidly in the form of road and railway tunnels, hydro-power tunnels/caverns power houses, storage structures of petroleum products, defense ammunitions, etc. Nowadays, many excavations for underground spaces are carried out in the mountain regions. The geology of these regions is extremely fragile and displays complex rock mass characteristics. Naturally, exact determination of rock mass properties becomes extremely difficult, if not impossible. On the other hand, uncertainties are present in tunnel support systems due to variation in lining thickness, strength of lining concrete, number of rock bolts, rock bolt diameter, etc. Hence, it is appropriate to analyze tunnels by considering the rock mass and support system properties as uncertain (Oreste, 2005).

The most popular method for uncertainty quantification is the Monte Carlo simulation (MCS) (Shinozuka, 1972; Rubinstein and Kroese, 1981). In this method, the response statistics are computed based on deterministic analysis at randomly generated sample points. Although easy to implement, a large number of sample points are required for obtaining satisfactory results. Due to this reason, use of this method is only limited to benchmarking newly developed tools for uncertainty quantification. A number of improvements to the conventional MCS, such as the Latin hypercube sampling (Seaholm et al., 1986; Iman et al., 1980; Park, 1994) and stratified sampling (Ericson, 1965), have also been proposed. All these methods are collectively known as simulation based approaches (SA). However, even the modified methods are often computationally expensive and hence, use of these methods is limited to small-scale problems only.

An interesting alternative to the SA is the non-simulation based approaches (NSA). Within the framework of NSA, the output response is approximated by using some series expansion. Popular NSAs include perturbation based approach (Wang and Mu, 2015; Gerstl, 1973; Wang and Qiu, 2014) and Newman's expansion (Gong et al., 2016). Unlike SA, NSAs are highly efficient. However, results obtained using these approaches are often erroneous, especially for problems involving higher orders of nonlinearity.

A third category of methods that has become quite popular over the last decade or so, is the surrogate based approaches. In this approach, a surrogate model that replicates the actual model is first formulated based on responses at some preselected training points. Once the surrogate model is formulated, response statistics are computed by

* Corresponding author.
E-mail address: csouvik41@gmail.com (S. Chakraborty).
[1] These authors have equal contribution.

http://dx.doi.org/10.1016/j.tust.2017.07.009
Received 13 March 2017; Received in revised form 11 July 2017; Accepted 14 July 2017

performing some suitable SA on the generated surrogate model. Surrogate models that are quite popular in literature include polynomial chaos expansion (Blatman and Sudret, 2011, 2010; Xiu and Karniadakis, 2002), Kriging (Arfaoui and Inoubli, 2013; Kaymaz, 2005; Biswas et al., 2016; Mukhopadhyay et al., 2017), radial basis function (RBF) (Bollig et al., 2012; Fang et al., 2005; Anicic et al., 2016), least square (Gavin and Yau, 2008; Lü and Low, 2011) and moving least square (Goswami et al., 2016; Lü et al., 2017) based response surface method, high-dimensional model representation (HDMR) (Ma and Zabaras, 2010; Chakraborty and Chowdhury, 2013; Chowdhury and Rao, 2009), etc. In this context, it is important to note that most of the surrogate models are either based on global approximation of the error norm (e.g. HDMR) or on local approximation of the error norm (e.g. Kriging). As a result, the surrogate models are either efficient (global approximation based surrogate models) or yield accurate results (local approximation based surrogate models). Hence, it is necessary to develop a surrogate model that is efficient as well as accurate.

This paper introduces a novel approach, referred to as hybrid polynomial correlated function expansion (H-PCFE) (Chatterjee et al., 2016; Chakraborty and Chowdhury, 2017b,c) for probabilistic analysis of tunnels. Compared to conventional surrogate models, H-PCFE has the following advantages:

- H-PCFE performs a bi-level approximation: first on a global scale by using polynomial correlated function expansion (PCFE) (Chakraborty and Chowdhury, 2015a,b, 2016a,b,c, 2017a; Chakraborty et al., 2016) and second on the local scale by using Kriging (Kaymaz, 2005; Biswas et al., 2016; Mukhopadhyay et al., 2017). As a consequence, the proposed approach is efficient as well as accurate.
- All the advantages of PCFE (Chakraborty and Chowdhury, 2015a,b, 2016a,b,c, 2017a; Chakraborty et al., 2016), such as the mean square convergence, optimality in Fourier sense are intrinsically present in H-PCFE.
- Unlike conventional surrogate models, H-PCFE is capable of treating both dependent and independent random variables without the need of any *ad hoc* transformation.

The primary objective of this work is to examine the performance of H-PCFE in uncertainty quantification of tunnel responses. It is to be noted that this is the first instance where such a hybrid approach has been utilized for uncertainty quantification of tunnel responses.

The rest of the paper is organized as follows: In Section 2, a generalised framework for uncertainty quantification using surrogate model has been discussed. Two popular surrogate models, namely RBF and Kriging have also been reviewed in this section. In Section 3 the fundamentals of H-PCFE have been discussed. Section 4 presents a unified framework for uncertainty quantification in tunnel responses using H-PCFE. In Section 5, two tunneling problems are presented to illustrate the performance of the proposed approach. Various case studies have also been reported in this section. Finally, the concluding remarks are presented in Section 6.

2. Surrogate modelling for uncertainty quantification

Suppose $\boldsymbol{X} = (X_1, X_2, ..., X_N)$ is a vector with N number of input variables (e.g., material properties, external load, support condition etc.), where $\boldsymbol{X} \in D \subset \Re^N$ ($\Re$ denotes real number) and y represents the output response (in this case the tunnel response). In actual problems, the relationship between the inputs and the output are often unknown and one often relies on expensive numerical techniques such as finite element method for computing the unknown response corresponding to a given set of inputs. Such an approach, although suitable for deterministic problems, is often impractical for uncertainty quantification due to its high computational demand. An interesting way to address this issue is to use surrogate models. The primary idea of surrogate model based approaches is to replace the actual costly model (often FE based) g by an efficient and accurate $\hat{g}$. All subsequent operations are carried out based on $\hat{g}$. An algorithm detailing the step-by-step procedure of surrogate based approach for uncertainty quantification is shown in in Algorithm 1.

Algorithm 1. Steps for performing uncertainty quantification using surrogate models

Initialize: Identify the input variables. Also identify the probability density function of the input variables.

1. Generate training points by using some suitable design of experiment scheme.
2. Obtain responses at the training points.
3. Train a surrogate model based on the inputs in step 1 and outputs at step 2.
4. Perform MCS on the generate surrogate model for quantifying the output uncertainty (probability density function and moments)

From above discussion, it is obvious that a surrogate model should have two desirable properties. Firstly, the number of training points required should be minimal as this directly influences the computational effort associated with a surrogate model. Secondly, results obtained using a surrogate model should be in close proximity of the actual model – without which, the response probability density function (PDF) and moments computed will be erroneous.

In the remainder of this section, two popular surrogate models (RBF and Kriging) have been reviewed. Note that these two surrogates are not the primary focus of this paper. Instead, they are used in this paper for demonstrating the superior performance of the proposed approach.

2.1. Radial basis function (RBF)

Radial basis function (RBF) is a surrogate model which is quite popular among researchers. RBF is often used to perform the interpolation of scattered multivariate data (Krishnamurthy, 2003; Hardy, 1971; Buhmann, 2000). The surrogate, $\hat{g}(\boldsymbol{X})$ appears in a linear combination of Euclidean distances, which may be expressed as

$$\hat{g}(\boldsymbol{X}) = \sum_{k=1}^{n} w_k \phi_k (\boldsymbol{X}, x_k) \tag{1}$$

where, n is the number of sampling points,[2] w_k is the weight determined by the least-squares method and $\phi_k(\boldsymbol{X}, x_k)$ is the k-th basis function determined at the sampling point x_k. Various symmetric radial functions $R_f(\boldsymbol{X})$ are used as basis function. Popular radial functions, $R_f(\boldsymbol{X})$ includes:

$$R_f(\boldsymbol{X}) = \exp\left(-\frac{(\boldsymbol{X}-c)^T(\boldsymbol{X}-c)}{r^2}\right) \quad \text{(For Gaussian)} \tag{2}$$

$$R_f(\boldsymbol{X}) = \sqrt{1 + \frac{(\boldsymbol{X}-c)^T(\boldsymbol{X}-c)}{r^2}} \quad \text{(For multi -quadratic)} \tag{3}$$

$$R_f(\boldsymbol{X}) = \frac{1}{\sqrt{1 + \frac{(\boldsymbol{X}-c)^T(\boldsymbol{X}-c)}{r^2}}} \quad \text{(For inverse multi -quadratic)} \tag{4}$$

$$R_f(\boldsymbol{X}) = \frac{1}{1 + \frac{(\boldsymbol{X}-c)^T(\boldsymbol{X}-c)}{r^2}} \quad \text{(For Cauchy)} \tag{5}$$

where c is the shift factor (normally considered to be the mean) and r is the normalising factor (generally considered to be the standard deviation). It is to be noted that RBF is not a regression technique. Rather, RBF may be broadly considered as an interpolation technique. This is

[2] Sample points and training points have been used synonymously in the context of surrogate model.