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The support characteristic curve for blocked steel sets in the convergence-confinement method of tunnel support design

Carlos Carranza-Torres[a,*], Maxwell Engen[b]

[a] Department of Civil Engineering, University of Minnesota Duluth, MN, USA
[b] J.F. Shea Construction, Indianapolis, IN, USA

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ABSTRACT

Construction of support characteristic curves is an important aspect of the implementation of the convergence-confinement method of tunnel support design. This paper presents a detailed analysis of the equations required to construct the support characteristic curve of a combined support system consisting of closed circular steel sets linked to the periphery of the tunnel by equally spaced prismatic wood blocks. This particular problem has been treated in the classical literature of underground excavation design in rock, although without providing details of the origin or validity of the equations. This paper fills this gap, providing the derivation and validation of all the equations required to construct the support characteristic curve for the combined support system. The paper shows that both the stiffness and maximum load that the combined support system can take increases as the angle between equally spaced wood blocks is decreased (or the number of blocks for the steel section is increased). In the limit, when the angle between wood blocks is assumed to be zero, the paper shows that the classical equations for construction of support characteristic curve for a steel set subjected to uniform radial load are recovered. The paper presents also a practical example illustrating the application of all equations needed to construct the support characteristic curve for the combined support system.

1. Introduction

The convergence-confinement method of tunnel support design is a convenient analytical-graphical procedure for estimating the required support, particularly in the early stages of the process of tunnel support design (see, for example, AFTES, 1978; Hoek and Brown, 1980; Fairhurst and Carranza-Torres, 2002). The method consists in constructing the *reaction curve* of the ground based on the planned diameter of the assumed circular tunnel, in-situ stresses and mechanical properties of the rock mass, together with the *support characteristic curve* of the intended type of support, based on selected mechanical properties (see, for example, Daemen, 1977; Hoek and Brown, 1980; Hoek, 2007; Hoek et al., 2008). From the construction of ground reaction and support characteristic curves, the ground pressure expected to act on the support can be determined as the intersection point of both curves, when plotted in a diagram representing the ground and support radial pressure, and the ground and support radial displacement.

This paper addresses the construction of the support characteristic curve, in particular, the one corresponding to circular steel sets linked to the periphery of the tunnel through equally spaced wood blocks. These types of support have been and are still used in construction of circular tunnels and shafts of relative to moderate size, not only for civil engineering but also for mining engineering purposes (see, for example, Proctor and White, 1946; Proctor and White, 1977). For a closed circular (annular) type of support considered in the convergence-confinement method, the characteristic curve is defined by the analytical elastic (or elastic-plastic) relationship between the radial load applied on the support and the resulting radial displacement. Obtaining the load-displacement relationship for the support involves solving a structural load-displacement problem. For example, for assumed elastic annular liners of relative large thickness (e.g., shotcrete or concrete liners with ratios of thickness to tunnel radius between 0.1 and 0.25), Lame's solution (Lamé, 1852; see also, Timoshenko and Goodier, 1970) has been used to derive the relationship between load and displacement (see, for example, Daemen, 1975; Brady and Brown, 2004; Hoek, 2007). Alternatively, similar equations can be obtained from the theory of *thin* and *thick* shells (see, for example, Flügge, 1967; Carranza-Torres and Diederichs, 2009). Analytical expressions to construct support characteristic curves for other types of support include those corresponding to rockbolts and cables, and steel sets used with and without wood blocks (see, for example, Hoek and Brown, 1980; Carranza-Torres and Fairhurst, 2000; Brady and Brown, 2004).

* Corresponding author.
E-mail address: carranza@d.umn.edu (C. Carranza-Torres).

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For the case of support consisting of steel sets installed with wood blocks, load-displacement relationships needed to construct the support characteristic curve have been originally presented by Daemen (1975). Later on, these equations have been transcribed in other publications dealing with the convergence-confinement method of tunnel support design (Hoek and Brown, 1980; Carranza-Torres and Fairhurst, 2000; Brady and Brown, 2004). The equations proposed by Daemen (1975), in turn, have been derived from the equations comprising the solution of loading of a circular elastic ring subjected to equally spaced radial point loads, as presented by Roark (1954)—see also Young and Budynas (2002). It turns out that the solution for radial displacements for the circular ring originally presented in Roark (1954) includes typographical or actual errors (the problem with the equations is discussed in Section 3). Furthermore, the equations proposed by Daemen (1975), which have been transcribed in the mentioned publications, do not comprise the full set of equations needed to determine the maximum load that the steel set and/or wood blocks are able to take to avoid material failure. Because of this, there exists the need to revise the solution to this problem and to present the correct and full set of equations needed to construct the support characteristic curve for steel sets installed with wood blocks (Brown, 2017; Fairhurst, 2017; Hoek, 2017).

The objective of this paper is therefore to present a comprehensive discussion on the construction of support characteristic curves for blocked steel sets in the convergence-confinement method of tunnel support design, providing the full set of equations and the details of their derivation and validation, together with an illustrative practical example of construction of the support characteristic curve.

2. Problem statement

The problem to be analyzed in this paper is shown in Fig. 1a. A section of circular tunnel is supported by a closed circular steel set that is linked to the periphery of the circular tunnel by equally spaced wood blocks. Steel set and wood blocks are assumed to be elastic and to fail plastically at specified values of compressive or tensile stresses, as will be discussed later on.

The support system comprised by steel set and wood blocks, which in following sections will be referred to as the *combined support*, is assumed to be equally spaced a distance, S, along the axis of the tunnel (see Fig. 1b). If the ground is assumed to transmit to the support a uniform pressure, p_R, throughout the length of tunnel, the load that each section of combined blocked steel set support takes is q_R; this load can be computed with the following equation (see Figs. 1a and b),

$$q_R = p_R S \tag{1}$$

The maximum radial convergence experienced by the combined support will be shown to occur at the location of the wood blocks (see points A and A' in Fig. 1a) and will be denoted as u_{sb}. One objective of this paper is to determine the linear stiffness of the combined support, K_{sb}, that according to the convergence-confinement method, is the ratio of the ground pressure, p_R, and the resulting maximum convergence of the combined support, u_{sb}.

Referring to Fig. 1a, the wood blocks are assumed to be equally spaced an angle θ_b. This angle can be computed based on the number of blocks for the steel set, n_b, with the following equation,

$$\theta_b = \frac{2\pi}{n_b} \tag{2}$$

The wood blocks are assumed to be rectangular prisms of height, t_b, and side lengths, w_b, and to be in full contact with the flange of the steel sets (see Figs. 1a and b); they are also assumed to be linearly elastic and perfectly plastic, with a Young's modulus, E_b, and an admissible compressive strength, $\sigma^b_{c,adm}$ (the admissible compressive strength is computed by dividing the yield compressive strength, $\sigma^b_{c,yield}$, by an appropriate factor of safety).

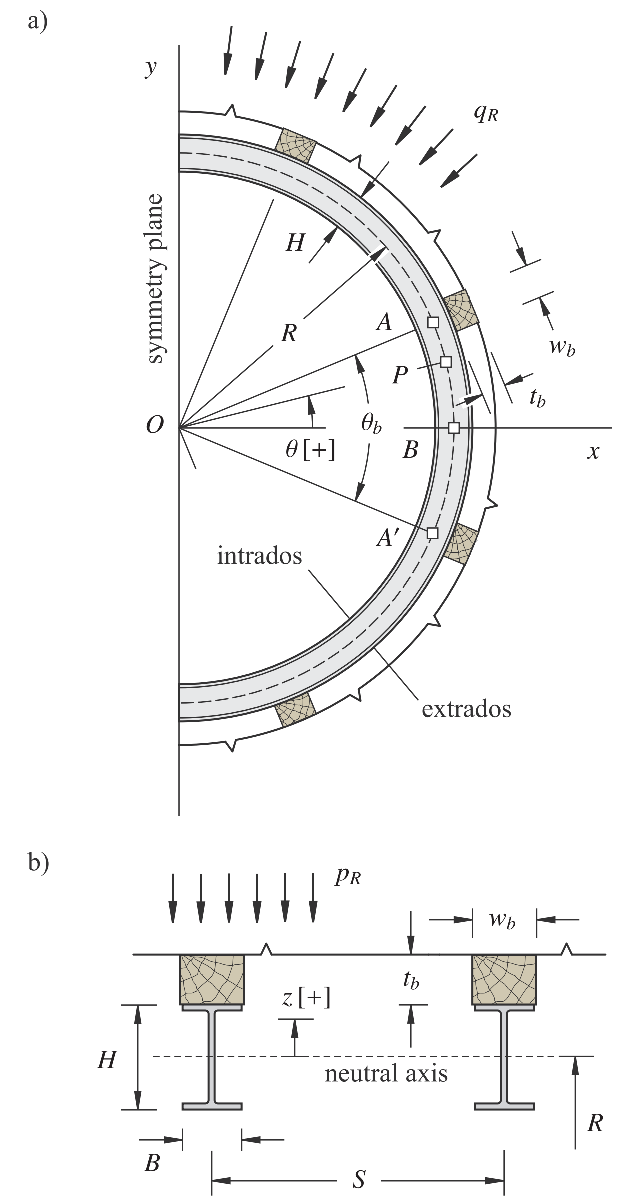

Fig. 1. (a) Tunnel support consisting of circular steel set linked to the ground by equally spaced wood blocks. (b) Detail of support along the axis of the tunnel.

Referring again to Figs. 1a and b, the steel set section is assumed to have a height H and width B. The radius of the closed circular steel set (measured from the tunnel center to the mid-height or neutral axis of the section) is denoted as R. The steel set is also assumed to be linearly elastic and perfectly plastic, with a Young's modulus, E_s, and admissible compressive and tensile strengths, $\sigma^s_{c,adm}$ and $\sigma^s_{t,adm}$, respectively (again, the admissible compressive and tensile strengths are obtained by dividing the yield compressive and tensile strengths, $\sigma^s_{c,yield}$ and $\sigma^s_{t,yield}$, respectively, by appropriate factors of safety). The steel section is assumed to have a cross sectional area, A_s, a moment of inertia, I_s, and a section modulus, S_s (the values assigned to the variables I_s and S_s consider that the steel section is positioned as indicated in Fig. 1b, i.e., with the top flange of the section being in contact with the wood block). As a reference, Table 1 (after CONSTRADO, 1983) lists values of the geometrical properties introduced above for some selected sections of universal beams and joists.

In the elastic solution for displacements of the steel set in Fig. 1a to be presented in later sections, two coefficients involving the stiffness and geometrical characteristics of the steel section will appear. The first coefficient, to be referred to as the *compressibility* coefficient, D, is