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Reliability analyses of underground openings with the point estimate method

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ABSTRACT

Reliability analyses usually need to treat a large amount of data to obtain satisfactory results. The most common reliability tool is Monte Carlo simulation (MCS). This technique demands a large number of evaluations of the mechanical behavior of the problem in question. Thus, MCS are less suitable for complex geotechnical numerical models like underground excavation stability. To overcome this limitation, approximation techniques are used in the field of structural reliability. The probability of failure P_f has been frequently estimated by means of the point estimate method (PEM) introduced by Rosenblueth (1975). Recently, some researches have focused on the enhancement of the PEM by improved sampling techniques and applying higher-order moments to approximate the probabilities of failure. This paper presents a comparison of the accuracy of three different schemes of point estimate methods: Rosenblueth (1975), Hong (1998) and Zhao and Ono (2000) to estimate the probability of failure of wall convergence of a circular tunnel and the face stability of a shallow tunnel by using higher moment approximation of the reliability index. Monte Carlo simulation (with or without importance sampling) approximations were performed to serve as a benchmark for evaluating the accuracy of the PEM methods. Results show that the classical second moment approximations are not suitable for tunnel wall convergence response, presenting errors larger than 250% in the estimate of reliability indices. The best results were obtained with Zhao and Ono's PEM with fourth moment approximation (FM-3) of the reliability index presenting errors below 20%. Regarding the face stability, all PEMs yielded accurate results with errors below 15%. Finally, the use of PEM is suggested only for preliminary analyses because of its general lack of accuracy.

1. Introduction

The quantification of the risk involved in underground projects has become increasingly important in recent years (BTS, 2003; Eskesen et al., 2004; ITIG, 2006). Consequently, estimating the probabilities of failure is of vital importance and an efficient, reliable and accurate method is needed to quantify the probability of failure of an underground work, even when there is no analytical tool for calculating it.

It is well known that the most reliable and general geotechnical analysis tools are numerical methods, since they allow the consideration of complex geometries and properties of geomaterials and are attractive because of the reliable responses they provide. On the other hand, depending on the complexity and quality of the analyses, numerical methods can be expensive in terms of computational demand. These characteristics make them quite unsuitable for evaluating the probability of failure by means of computationally demanding techniques like the crude Monte Carlo simulation (MCS), or even the importance sampling Monte Carlo simulation (ISMCS), among others. Therefore, it is essential to reduce the computational effort and to perform reliability calculations in a more efficient fashion. This issue has been addressed in the structural reliability field, mainly by means of analytical approximation methods and to a lesser extent by sampling-based methods.

Analytical approximation methods use optimization algorithms to find the most probable array of variables that could induce the failure of a structural system. This array of variables is known as the design point (DP). Once the DP is found, the probability of failure can be estimated by linear, quadratic, spectral or higher-order

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Nomenclature

List of symbols

$\alpha_{i,3}$, $\alpha_{i,4}$	skewness and kurtosis of the *i*th random variable
α_{3g}, α_{4g}	skewness and kurtosis of $g(\mathbf{X})$
α_{3gi}, α_{4gi}	skewness and kurtosis of $g_i(\mathbf{X})$
β	reliability index
β_{SM}, β_{TM}, β_{FM}	second-moment, third-moment and fourth-moment approximations of β
$\mathrm{E}[\cdot]$	statistical expectation
$g(\mathbf{X})$	performance function
$g_i(\mathbf{X})$	performance function at $\mathbf{U}_i$
m	number of concentration points
$M_{k,gi}$	*k*th raw moment of $g_i(\mathbf{X})$
μ_i, σ_i	mean and standard deviation of the *i*th random variable
μ_g, σ_g	mean and standard deviation of $g(\mathbf{X})$
μ_{gi}, σ_{gi}	mean and standard deviation of $g_i(\mathbf{X})$
n	number of random variables
$\rho_{\mathbf{i,j}}$	correlation coefficient among the ith and jth random variables
$\mathbf{R_X}$, $\mathbf{R_Z}$	correlation matrix in $\mathbf{X}$ and $\mathbf{Z}$ space
$s_{i,p}$	indicator function that adopts +1 to sample above and -1 to sample below the mean used in Rosenblueth PEM
$\mathrm{T}^{-1}(\cdot)$	Rosenblath transformation
$\mathbf{U}_i$	vector of random variables when the only random variable that changes is u_i, and the other random variables are set at their mean value image in the Gaussian space
$\mathbf{u}_{i,p}$	*p*th concentration point of the *i*th random variable in the Gaussian space
w_{cp}	weighting factor of the *cp*th concentration point used in Rosenblueth PEM
$w_{i,p}$	*p*th concentration point of the *i*th random variable
$\mathbf{X}_{cp}$	*cp*th concentration point used in Rosenblueth PEM
$\mathbf{X}_{i,p}$	*p*th concentration point of the *i*th random variable
$x_{i,p}$	*p*th concentration point component corresponding to *i*th random variable
$\xi_{i,p}$	position parameter of the *p*th concentration point of the *i*th random variable used in Hong PEM

approximations (Breitung, 1984; Cai and Elishakoff, 1994; Hasofer and Lind, 1974; Köylüoğlu and Nielsen, 1994; Rackwitz and Fiessler, 1978; Tvedt, 1983; Zhao and Ono, 1999a,b; Zhao et al., 2002). These techniques have been successfully used to perform the quantification of the reliability of underground structures with numerical models by means of the Direct Coupling Approach (Napa-García, 2015; Napa-García et al., 2016).

Sampling-based methods include simulation methods and the point estimate method (PEM). Simulation methods like MCS and ISMCS are not efficient when dealing with numerical analyses (Park et al., 2012) because of the large number of response evaluations necessary to obtain satisfactory results. On the other hand, the PEM is a straightforward method that employs only a few samples to estimate the first moments of a function of random variables (RVs).

2. Point estimate method

The PEM was initially introduced by Rosenblueth (1975). In the first version, three cases were considered: (1) univariate function with mean, variance and skewness known, (2) univariate function of an approximately Gaussian RV, i.e. null skewness and (3) multivariate function of approximately Gaussian correlated RVs. The Rosenblueth PEM has been used to estimate the behavior of implicit numerical and analytical responses of underground structures (Park et al., 2013, 2012) and in geotechnical engineering in general (Christian and Baecher, 1999, 2001, 2002; Christian et al., 1994; Esterhuizen, 1990; Miller et al., 2004). Christian and Baecher (1999) emphasized that the limitation of the Rosenblueth PEM lies in the fact that it is not able to represent moments higher than the second, particularly when the response is not well represented by a third-order polynomial and when the coefficients of variation (CVs) of the RVs are large.

In recent years, improvements in PEM sampling techniques have been suggested and the accuracy of the higher-order moments has been refined (Hong, 1998; Lin and Li, 2013; Zhao and Ono, 2000, 2001, 2004). Consequently, the accuracy of reliability estimation based on discrete sampling is comparable with that of the analytical methods for simple performance functions. Lin and Li (2013) pointed out some restrictions and limitations in the use of the PEM as a reliability tool. Also, they observed that only Hong (1998) used a strict Taylor's series expansion. Zhao and Ono (2000) presented a new point estimate that differed from Rosenblueth's procedure in the number and location of the samples. They used a 5n/7n sampling scheme based on the Gaussian-Hermite integration briefly described by Rosenblueth (1975). The advantage of Hong's method as well as Zhao and Ono's method is that the sampling process takes into account the skewness and kurtosis of the input RVs. These characteristics were only available in the first univariate Rosenblueth PEM.

As this paper is focused on the Rosenblueth, Hong and Zhao and Ono PEMs, a brief explanation of the sampling techniques as well as the method for estimating the moments of the performance function used by all the methods is now presented.

2.1. Rosenblueth 2^n PEM

Rosenblueth (1975) introduced the PEM to approximate the moments of the response of a function $g(\mathbf{X})$ of RVs (one or more than one). As described above, the method uses only the first two moments of the RVs for multivariate functions. The sampling process consists in selecting one point in every hyperquadrant of the RV space. The points selected are symmetrical around the mean value, i.e. one sample above and the other below the mean. Each sampling point has an associated weight which is determined by the number of RVs and their one-to-one correlation. Christian and Baecher (1999) presented a generalization of the formulation as shown below.

Concentration point $\mathbf{X}_{cp}$ is a combination of the n components of the type:

$$x_{i,p} = \mu_i + s_{i,p} \cdot \sigma_i \tag{1}$$

where $x_{i,p}$ is the component of the concentration point corresponding to the *i*th RV, μ_i and σ_i are mean and standard deviation of the *i*th RV, and $s_{i,p}$ is a function that adopts +1 when the value of the *i*th RV is higher than the mean value and -1 when the value is lower than the mean value.

Weighting factors are:

$$w_{cp} = \frac{1}{2^n}\left[1 + \sum_{i=1}^{n-1}\sum_{j=1}^{n} s_{i,p} \cdot s_{i,p} \cdot \rho_{i,j}\right] \tag{2}$$

where $\rho_{i,j}$ is the correlation coefficient between the *i*th and *j*th RVs.

Finally, the *k*th raw moment of $g(\mathbf{X})$ is calculated as

$$\mathrm{E}\left[\{g(\mathbf{X})\}^k\right] \approx \sum_{cp=1}^{2^n} w_{cp} \cdot \left[g(\mathbf{X}_{cp})\right]^k \tag{3}$$