Tunnelling and Underground Space Technology 64 (2017) 164–176

Contents lists available at ScienceDirect

Tunnelling and Underground Space Technology

journal homepage: www.elsevier.com/locate/tust

Prediction of tunnel lining forces and deformations using analytical and numerical solutions

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ARTICLE INFO

Article history:
Received 15 October 2015
Received in revised form 5 December 2016
Accepted 23 January 2017

Keywords:
Lining forces
Analytical solution
Numerical analysis
Tunnel design
Soil-structure interaction
Arching effect

ABSTRACT

Structural design of linings requires a reliable prognosis of lining forces and deformations. In engineering practice, both analytical and numerical solutions are popular to be employed to predict the system behavior. This paper employs the commonly accepted analytical solutions to calculate the lining forces and deformations for both shallow and deep tunnels, the results are compared to the numerical results for corresponding equivalent boundary conditions, initial conditions and identical material properties. Afterward, more sophisticated constitutive models for soil/structure elements in conjunction with more realistic construction aspects are taken into account. The comparison of the results of analytical and numerical solutions highlights the differences between these two well accepted methods as well as the effect of considering realistic features in numerical simulations. Moreover, the lining forces and deformations obtained from plain strain condition are compared to the 3D numerical results. The results show that the analytical bedding model is able to reasonably predict the lining behavior for both shallow and deep tunnels even if the soil is assumed to be an elastic material. In numerical solutions, lining forces and deformations depend to a large extent on the applied soil constitutive model and construction method. The face support pressure, backfill grouting and arching effect cannot be captured appropriately in plain strain condition, which leads to the discrepancy between the model responses obtained from 2D numerical/analytical solutions and realistic 3D simulations.

1. Introduction

Proper design of lining segments plays a pivotal technical and economical role in mechanized excavation for both shallow and deep tunnels. For the conventionally driven open face tunneling technique, stress release of soil domain after excavation has an important impact on the structural design of concrete lining. Nowadays, tunnel boring machines (TBM) have been widely applied in tunnel construction, especially in urban areas where the most important aim is to minimize the soil deformation. The behavior of lining segments is affected by the complex construction features, for example the sequential excavation process and backfill grouting. Therefore, developing a framework to accurately predict the lining forces and deformations before tunnel construction is essential for the purpose of structural safety and optimum design. Additionally, the cost of tunnel construction depends to a large extent on the cost of lining, this also contributes to the importance of structural design of linings on short and long terms stability of the tunnel.

Axial forces, bending moments and radial displacements are the most significant responses of lining segments during construction, which strongly depend on the confining pressure due to the surrounding soil stresses. As soil deformation and soil-lining interaction induce the variation of soil stresses applied on lining, analysis of tunnel lining and its interaction with soil becomes even more complex because of the dependence of such interaction on the construction technology and schemes (El-Nahhas et al., 1992).

In order to reliably predict the lining forces and deformations, finite element method (FEM) analysis has become a popular tool which can simulate staged construction procedures and reproduce the soil and structure behavior by using appropriate constitutive models. In engineering practice, the numerical modeling often relies on the two dimensional (2D) analysis, since it is straightforward and cost-effective. For the numerical simulation of mechanized tunneling process in plain strain condition, it

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http://dx.doi.org/10.1016/j.tust.2017.01.015
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normally takes into account the tunnel construction process including TBM excavation, backfill grouting and lining installation as well as the soil-lining interaction. Oreste (2007) developed a special code within the FEM framework using hyperstatic reaction method to consider the actual geometry of the lining support and the horizontal loads that are different from the vertical ones, it is therefore able to analyze the mass-structure interaction in detail. Möller and Vermeer (2008) applied FEM to simulate the conventionally driven Steinhaldenfeld tunnel and Heinenoord slurry shield tunnel, and studied the influences of constitutive model and applied construction method on lining forces and ground deformation. Zhang et al. (2015) analyzed the influence of multi-layered soil formation on tunnel lining behavior by employing FEM, and the results show a good agreement between the numerical model responses and the real measurements.

In the realistic tunneling process, the excavation procedure changes the primary stress field at the tunneling face (ITA, 1988). Furthermore, the soil stresses on lining segments are influenced by the sequential excavation process and 3D arching effect of the soil towards the end of tunnel. Despite of the popularity of 2D numerical analysis, its deficiencies, that face support in front of TBM, the sequential excavation process and the inclination of the tunnel cannot be modeled, are inevitable. Within this framework, the use of 3D FEM analysis is essential if one wants to correctly evaluate the influence of staged excavation process on lining structure responses. Hudoba (1997) studied how the lining structures react under static loading of the surrounding soil during tunneling process using both 2D and 3D computing models. Galli et al. (2004) modeled tunnel excavation and lining installation in both 2D and 3D models, they showed that 3D discretization of soil-tunnel system is essential to analyze the soil deformation and stresses in the lining elements.

Although FEM analysis is a powerful tool in simulations of engineering problems, uncertainty in model responses is unavoidable due to the complex tunnel construction procedures, limitations of employed FEM techniques and the insufficiency of constitutive models themselves. Additionally, it costs many resources to obtain the input parameters to be used in numerical model, while running the sophisticated model is time-consuming, especially for complex 3D tunneling model. For the preliminary design of lining segments, analytical solutions can be used to give a good insight into the dominant processes. The analytical solutions are developed on the basis of 2D idealization of tunnel construction. An overview of the contributions to analytical solutions is given in Table 1. The list and the following brief discussion are, of necessity, incomplete. Schmid (1926) was probably the first who proposed an analytical solution for thick lining segments in contact with elastic soil. A thinner lining is assumed by Voellmy (1937), however, his results do not comply with the equilibrium of forces since they neglected the tangential components of soil pressure. In 1964, Schulze and Duddeck (1964a) published a complete and closed solution which was applicable for shallow tunnels. After that, Windels (1967) proposed a complete solution on the circular tunnel in an elastic soil domain by also taking into account the geometric nonlinearity and an approximation for the lining deformations. In 1982, an exhaustive and theoretically complete solution was finally achieved by Ahrens et al. (1982). Additionally, Molins and Arnau (2011) and Arnau and Molins (2011) conducted experimental and analytical study of the structural response of segmental lining based on an in situ loading test. Their approach is able to asses a realistic consideration of the soil-structure interaction.

Table 1
Brief overview of the contributions to structural design models of linings.

Reference	Description
Schmid (1926)	First to analyze the elastic continuum with considering the soil-lining interaction
Voellmy (1937)	The continuum model, omission of the tangential components of soil pressure
Bull (1944)	The bedding model for shallow tunnel, tedious calculation
Engelbreth (1961)	The continuum model with closed form
Schulze and Duddeck (1964a)	The bedding model with complete and closed solution
Windels (1967)	The continuum model with complete solution
Ahrens et al. (1982)	The exhaustive and theoretically complete solution
Bakker (2003)	Unidimensional model without considering the soil-lining interaction
Kim and Eisenstein (2006)	Using correction factors considering the non-linear ground behavior

In cases where the lining responses from analytical and numerical solutions differ expensively, Behnen et al. (2015) pointed out that a design strategy based on simple or comprehensible analytical models in combination with the valuable knowledge of experienced engineers should be preferred instead of relying on the complex computational models. In this study, different commonly accepted analytical methods are compared firstly in order to evaluate their applicability. Afterward, the FE-models for tunneling simulation and corresponding hierarchical modeling strategies are illustrated. By doing so, the results of analytical and numerical solutions are compared for equivalent boundary conditions, initial conditions and identical material properties. Subsequently, more sophisticated constitutive models for soil/structure elements in conjunction with more realistic construction aspects are taken into account. The comparison of the results of analytical and numerical solutions highlights the differences between these two well accepted methods as well as the effect of considering realistic features in numerical simulations. Additionally, the difference between lining forces and deformations calculated via 2D and 3D FE-models is also discussed.

2. Analytical solutions

An overview of different contributions to structural design models for tunnel linings is given in Table 1. General agreement of these analytical models lies on the following basic assumptions: (1) the analytical solutions are sufficient to consider only a cross-section, which means plain strain condition is assumed; (2) the cross section of the tunnel is assumed to be circular; (3) the soil stresses on the lining segments are assumed to be equal to the primary stresses in the undistributed ground; (4) there is a bond between the lining and the ground, it takes into account the soil-lining interaction; and (5) the material behavior of soil and lining is generally assumed to be elastic. In Germany two main categories of structural models for mechanized tunneling have been proven and established for usual applications: the continuum models (see Fig. 1(a) and (b)) preferred for deep tunnels and the bedding models (see Fig. 1(c)) preferred for shallow tunnels.

Commonly, the analytical continuum model consists of a homogenous elastic circular ring embedded in a plane 2D-continuum (see Fig. 1(a)). Herein, the idealized primary stress state is obtained from equilibrium of vertical and horizontal forces induced by earth pressures. The vertical component of lining load (σ_v) is modeled as an uniformly distributed load on top and bottom of the tunnel. This lining load is depth independent and determined based on the soil stress at the depth of tunnel axis. The horizontal load (σ_h) is also applied as a constant pressure, its magnitude is defined as the vertical earth pressure multiplied by the coefficient of lateral earth pressure at rest (K_0), namely $\sigma_h = K_0\sigma_v$. To compute the internal forces on lining, it is necessary