Journal of Mathematical Psychology 75 (2016) 137–149

Contents lists available at ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmp

What are we estimating when we fit Stevens' power law?

Michele Bernasconi [a], Raffaello Seri [b,c,*]

[a] Dipartimento di Economia, Università "Ca' Foscari", Cannaregio 873, I-30121 Venezia, Italy
[b] Dipartimento di Economia, Università dell'Insubria, Via Monte Generoso 71, I-21100 Varese, Italy
[c] Center for Nonlinear and Complex Systems, Università dell'Insubria, Via Valleggio 11, I-22100 Como, Italy

HIGHLIGHTS

- Estimates in Stevens' power laws often display sensitivities to experimental design.
- These so-called 'contextual effects' concern range, location and averaging.
- This paper links them with the separable representation model of Luce and Narens.
- Theoretical results are illustrated using data from papers of R. Duncan Luce.

ARTICLE INFO

Article history:
Available online 14 June 2016

Keywords:
Separable representations
Stevens' model
Psychophysical experiments

ABSTRACT

Estimates of the Stevens' power law model are often based on the averaging over individuals of experiments conducted at the individual level. In this paper we suppose that each individual generates responses to stimuli on the basis of a model proposed by Luce and Narens, sometimes called separable representation model, featuring two distinct perturbations, called psychophysical and subjective weighting function, that may differ across individuals. Exploiting the form of the estimator of the exponent of Stevens' power law, we obtain an expression for this parameter as a function of the original two functions. The results presented in the paper help clarifying several well-known paradoxes arising with Stevens' power laws, including the range effect, i.e. the fact that the estimated exponent seems to depend on the range of the stimuli, the location effect, i.e. the fact that it depends on the position of the standard within the range, and the averaging effect, i.e. the fact that power laws seem to fit better data aggregated over individuals. Theoretical results are illustrated using data from papers of R. Duncan Luce.

1. Introduction

A large part of the success of modern psychophysics is certainly due to the versatility and simplicity of Stevens' psychophysical law (Stevens, 1946, 1951, 1957): namely, the notion that sensation magnitude can be described as a power function of stimulus intensity. This idea has in particular been popularized by Stevens through the application of different direct measuring methods able to reveal the law and to provide estimates of the exponent of the power model in several sensory domains (see the posthumous book by Stevens, 1975, for a comprehensive survey).

In possibly the simplest of Stevens' direct measuring techniques, known as ratio magnitude estimation, a subject is asked to compare two stimuli, a comparison stimulus d_1 and a reference standard d_2, and to state in what proportion p the stimuli are with respect to each other. According to the power law, the following ratio scale of subjective intensities holds:

$$\left(\frac{d_1}{d_2}\right)^{\beta} = p. \tag{1}$$

Hence, the exponent β of the law can be easily estimated from a series of trials, in which d_1 varies between trials while the standard d_2 can both be kept constant or let vary as well (see below). Magnitude estimation and the complementary approach of magnitude production, in which the standard d_2 is given and the subject is asked to adjust d_1 to a prescribed ratio p, are still widely used (recent surveys, examples, discussions in several fields of psychophysics in Fagot, 2011; Lawless & Heymann, 2010; Lim, 2011; Masin, 2014; Shofner & Selas, 2002).

* Corresponding author at: Dipartimento di Economia, Università dell'Insubria, Via Monte Generoso 71, I-21100 Varese, Italy.
E-mail addresses: bernasconi@unive.it (M. Bernasconi), raffaello.seri@uninsubria.it (R. Seri).

http://dx.doi.org/10.1016/j.jmp.2016.04.007

Notwithstanding the ample and enduring success, it is however also well-known that Stevens' power law suffers from both empirical and theoretical shortcomings.

Under the empirical perspective, an ever growing experimental literature has documented several inconsistencies and puzzles of the law (Luce & Krumhansl, 1988, provide a classical survey; Zwislocki, 2009, Chapter 2, a recent one). Among them, a list of studies conducted since the early development of the approach documented that the exponent β of Stevens' law is sensitive to the experimental parameters and design of the investigations. In Section 2 we provide an account of the above earlier evidence.

Under the theoretical perspective, Stevens never provided a formalized theory of measurement. Successive scholars have worked in the tradition of the representational theory of measurement (in the three classic volumes of *Foundations of Measurement*, Vol. I by Krantz, Luce, Suppes, & Tversky, 1971, Vol. II by Suppes, Krantz, Luce, & Tversky, 1989, and Vol. III by Luce, Krantz, Suppes, & Tversky, 1990) to give more proper mathematical and philosophical foundations to the notion of psychophysical measurement (earlier works in Krantz, 1972, Luce, 1959, 1990, Shepard, 1981, and several others quoted in Luce, 1996, and Luce & Krumhansl, 1988).

Mainly, a recurrent criticism of mathematical psychologists was that neither the power law nor Stevens' method of direct estimation were derived from primitive behavioral conditions, or axioms, which could be independently expressed and tested. The development of the representational theory of measurement drawn in the three volumes of the *Foundations* represented in such a respect a revolution (see, e.g., Steingrimsson, 2016, in particular the Introduction).

An important more recent achievement developed in this stream of literature includes the axiomatization of various novel theories which comprehend Stevens' model as a special case (Augustin, 2006, 2010; Luce, 2002, 2004, 2008; Narens, 1996, 2002, 2006). Following a terminology introduced by Luce (2002), we say formally that a psychological scale of subjective intensities can be represented in a *separable form* if there exist a *psychophysical function* ψ and a *subjective weighting function* W such that p is in the following relation with d_1 and d_2:

$$\frac{\psi\left(d_1\right)}{\psi\left(d_2\right)} = W\left(p\right). \tag{2}$$

Eq. (2) incorporates the notion that various and independent distortions may occur both in the assessment of subjective intensities and in the determination of subjective ratios. Stevens' power model in Eq. (1) is obviously a particular case of separable representation, holding when W can be represented as the identity function and ψ is a power function.

Several experiments have given substantial support to separable representations, but not to the restrictions implied by the power law (Augustin & Maier, 2008; Bernasconi, Choirat, & Seri, 2008; Ellermeier & Faulhammer, 2000; Steingrimsson, 2009; Steingrimsson and Luce, 2005a, 2007; Zimmer, 2005).

Narens (1996) has obtained Eq. (2) in an article in which he formalized Stevens' magnitude methods in terms of axiomatic measurement theory.[1] Luce (2002, 2004, 2008) has axiomatized Eq. (2) as a special case of a global psychophysical theory of intensity perception. The theory has been shown to be general enough to be extended theoretically in many directions (Luce, 2012a, 2012b, 2013; Luce, Steingrimsson, & Narens, 2010). In the course of the paper we will give some accounts of the properties predicted and of the results obtained. We will also refer to some earlier nonaxiomatic approaches that have considered forms similar to separable representations providing experimental results in their support (as in, e.g., Birnbaum, 1980; Birnbaum & Elmasian, 1977 and Birnbaum & Mellers, 1978).

Given the inconsistencies against Stevens' power law anticipated above and described in detail below, a natural question arises about what are the many people that keep fitting the power law actually estimating.[2] This is what we clarify in this paper. In order to conduct the analysis we use an empirical working model developed from Bernasconi et al. (2008). In that previous paper, we estimated several variants of separable representation models and we saw which one performed best. In order to do so, we rewrote the model in Eq. (2) using a log–log transformation, we added a Fechnerian error term, and then expanded the log-transformed functions in polynomials of the various separable models. In this paper we develop a similar model, but do not use polynomial expansions. This allows for greater flexibility, which we use to reinterpret the parameters associated with the power law. In particular, by exploiting the form of the estimator of β, we obtain an expression for this parameter that we use to predict several facts, documented in the earlier literature, on the sensitivity of β to the experimental design, including the so called 'range effect', 'location effect' and 'averaging effect', which we illustrate with data digitized from two classical experiments of Green and Luce (1974) and Luce and Mo (1965).

We start in Section 2 with a review of the earlier evidence on Stevens' power law. In Section 3 we present the empirical working model and its relations to the literature. Results are in Sections 4 and 5. In Section 4 we apply the model to the study of ratio magnitude estimation with a standard for a single individual and we provide a theoretical account of range and location effects. In Section 5 we show how conducting the analysis with data averaged across individuals, rather than at the individual level, leads to the averaging effect. Section 6 concludes with a summary of the main findings and a discussion of extensions and implications of the approach. The proofs are gathered in the Appendix.

2. Contextual effects

In the following we are going to present a series of phenomena arising in magnitude estimation, as well as in related forms of scaling, that describe some deviations with respect to Stevens' power law that are often observed in the data. These are sometimes called 'context' or 'contextual effects' in the literature.

One of the most commonly observed contextual effects is the so-called 'range effect', i.e. the fact that for larger ranges of δ_1 (here and in the following $\delta_i = \ln d_i$) we expect β to be smaller. Early evidence on the effect was observed by Engen (1956), Engen and Levy (1958) and Künnapas (1960, 1961). Various experiments conducted in the following years confirmed the same conclusion (surveys and examples in, e.g., Bonnet, 1969a, 1969b; Teghtsoonian, 1971, 1973; Vincent, Brown, Markley, & Arnoult, 1968). Poulton (1968) reviews the literature up to that date and states that the range of stimuli "alone accounts for about $\frac{1}{3}$ of the variance in S. S. Stevens' table of exponents" (p. 1). It should also be noted that most of the previous evidence is based on data grouped over individuals, while for individuals the situation is less clear. The individual-level results in Pradhan and Hoffman (1963) do not seem to support this contextual effect (see, however, below for more discussion). On the other hand, always at the individual

[1] Therein he also asserts something that is slightly more general than our Eq. (1), namely $W(p) = p^k$ with $k > 0$, and $W(1) = 1$ (more detail in Section 3 below and in Steingrimsson & Luce, 2007, in particular their Section 2.1.1).

[2] For example, in addition to the surveys quoted above, in a systematic literature search Kornbrot (2014) identifies 193 items with "magnitude estimation" in the title published between 2000 and 2013 and remarks that just two studies have estimated psychophysical functions more general than Stevens' power law.