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An experimental test of reduction invariance

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HIGHLIGHTS

- Prelec's compound-invariant function is widely used to model probability weighting.
- Luce characterized this family by a tractable condition: reduction invariance.
- This paper tests reduction invariance in an experiment.
- Our data supported reduction invariance.
- Evidence on reduction of compound gambles was mixed.

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ABSTRACT

Prelec's (1998) compound-invariant family provides an appealing way to model probability weighting and is widely used in empirical studies. Prelec (1998) gave a behavioral foundation for this function, but, as pointed out by Luce (2001), Prelec's condition is hard to test empirically. Luce proposed a simpler condition, reduction invariance, to characterize Prelec's weighting function that is easier to test empirically. Luce pointed out that testing this condition is an important open empirical problem. This paper follows up on Luce's suggestion and performs an experimental test of reduction invariance. Our data support reduction invariance both at the aggregate level and at the individual level where reduction invariance was the dominant pattern. A special case of reduction invariance is reduction of compound gambles, which is often considered rational and which characterizes the power weighting function. Reduction of compound gambles was rejected at the aggregate level even though 60% of our subjects behaved in line with it.

1. Introduction

Probability weighting is an important reason why people deviate from expected utility (Fox & Poldrack, 2014; Luce, 2000; Wakker, 2010). Prelec (1998) proposed a functional form for the probability weighting function that is widely used in empirical research and that usually gives a good fit to empirical data (Chechile & Barch, 2013; Sneddon & Luce, 2001; Stott, 2006).

Although other functional forms have also been used (e.g. Currim & Sarin, 1989; Goldstein & Einhorn, 1987; Karmarkar, 1978; Lattimore, Baker, & Witte, 1992 and Tversky & Kahneman, 1992), Prelec was the first to give an axiomatic foundation for a form of the probability weighting function.[1] His central condition, compound invariance (defined in Section 2), is, however, complex to test empirically as it involves four indifferences and may be subject to error cumulation. To the best of our knowledge, it has not been tested yet.

Luce (2001) proposed a simpler condition, reduction invariance. Luce (2000, p.278) identified testing reduction invariance as an important open empirical problem. The purpose of this paper is to follow up on Luce's suggestion and to test reduction invariance in an experiment. Our data support the validity of reduction invariance. At the aggregate level, we found evidence for the condition and at the individual level it was clearly the dominant pattern.

A special case of reduction invariance is the rational case of reduction of compound gambles, which implies that the probability weighting function is a power function. Our data on reduction of compound gambles are mixed. At the aggregate level reduction of

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[1] For a more recent axiomatic analysis of probability weighting see Diecidue, Schmidt, and Zank (2009).

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compound gambles was clearly violated. However, 60% of our subjects behaved in line with it. The subjects who deviated, did so systematically and found compound gambles more attractive than simple gambles.

2. Background

Let (x, p) denote a *gamble* which gives consequence x with probability p and nothing otherwise. Consequences can be pure, such as money amounts, or they can be a gamble (y, q) where y is a pure consequence. The set of pure consequences is a nonpoint interval $\mathcal{X}$ in $\mathbb{R}^+$ that contains 0. Preferences $\succcurlyeq$ are defined over the set $\mathcal{C}$ of gambles. We identify preferences over simple gambles (x, p) from preferences over $((x, p), 1)$ and preferences over consequences x from preferences over $(x, 1)$.

A function U *represents* $\succcurlyeq$ if it maps gambles and pure consequences to the reals and for all gambles (x, p), (x', p') in $\mathcal{C}$, $(x, p) \succcurlyeq (x', p') \Leftrightarrow U(x, p) \geq U(x', p')$. If a representing function U exists then $\succcurlyeq$ must be a *weak order*: transitive and complete. The representing function U is *multiplicative* if there exists a function $W : [0, 1] \to [0, 1]$ such that:

i. $U(x, p) = U(x) W(p)$.
ii. $U(0) = 0$ and U is continuous and strictly increasing.
iii. $W(0) = 0$ and W is continuous and strictly increasing.

The functions U and W are unique up to different positive factors and a joint positive power: $U \to a_1 U^b$ and $W \to a_2 W^b$, $a_1, a_2, b > 0$. This uniqueness implies that we can always normalize W such that $W(1) = 1$.[2] Luce (1996, 2000) and Marley and Luce (2002) gave preference foundations for the multiplicative representation. A central condition in these results is consequence monotonicity, which we also assume here.[3]

The multiplicative representation is general and contains many models of decision under risk as special cases. Examples are expected utility, rank- and sign-dependent utility (Quiggin, 1981, 1982), prospect theory (Tversky & Kahneman, 1992), disappointment aversion theory (Gul, 1991), and rank-dependent utility (Luce, 1991; Luce & Fishburn, 1991, 1995).

Prelec (1998) axiomatized the following family of weighting functions:

Definition 1. $W(p)$ is *compound-invariant* if there exist $\alpha > 0$ and $\beta > 0$ such that $W(p) = \exp(-\beta(-\ln p)^{\alpha})$.

Prelec's compound-invariant weighting function has several desirable properties. First, it includes the power functions $W(p) = p^{\beta}$ as a special case. The class of power weighting functions is the only one that satisfies *reduction of compound gambles*, which is often considered a feature of rational choice:

$$((x, p), q) \sim (x, pq).$$

A second advantage of the compound-invariant family is that for $\alpha < 1$, it can account for inverse S-shaped probability weighting, which has commonly been observed in empirical studies (Fox & Poldrack, 2014; Wakker, 2010). Finally, the parameters α and β have an intuitive interpretation (Gonzalez & Wu, 1999). The parameter α reflects a decision maker's sensitivity to changes in probability, with higher values representing more sensitivity, while β reflects the degree to which a decision maker is averse to risk, with higher values reflecting more aversion to risk.

The compound-invariant family of weighting functions satisfies the following condition:

Definition 2. Let N be any natural number. N-*compound invariance* holds if $(x, p) \sim (y, q)$, $(x, r) \sim (y, s)$, and $(x', p^N) \sim (y', q^N)$ imply $(x', r^N) \sim (y', s^N)$ for all nonzero consequences x, y, x', y' and nonzero probabilities p, q, and r.

Compound invariance holds if N-compound invariance holds for all N. Prelec (1998) showed that if compound invariance is imposed on top of the multiplicative representation then $W(p)$ is compound-invariant. Bleichrodt, Kothiyal, Prelec, and Wakker (2013) showed that compound invariance by itself implies the multiplicative representation and, consequently, that the assumption of a multiplicative representation is redundant.

Compound invariance is difficult to test empirically. It requires four indifferences and elicited values appear in later elicitations, which may lead to error cumulation. For example, we could fix x, p, q, r, and x'. The first indifference would then elicit y, the second s, and the third y'. If each of these variables is measured with some error then this will affect the final preference between (x', r^N) and (y', s^N).

To address the problem of error cumulation, Luce (2001) proposed a simpler condition.

Definition 3. Let N be any natural number. N-*reduction invariance* holds if $((x, p), q) \sim (x, r)$ implies $((x, p^N), q^N) \sim (x, r^N)$ for all nonzero consequences x and nonzero probabilities p, q, and r.

Reduction invariance holds if N-reduction invariance holds for all N. Reduction invariance is easier to test than compound invariance as it requires only two indifferences. Luce (2001, Proposition 1) showed that if N-reduction invariance for $N = 2, 3$ is imposed on top of the multiplicative representation then the weighting function $W(p)$ is compound-invariant. To the best of our knowledge, Bleichrodt et al.'s (2013) result cannot be generalized to reduction invariance and the multiplicative representation still has to be assumed in this case.

[2] Aczél and Luce (2007) analyzed the case where $W(1) \neq 1$ to model non-veridical responses in psychophysical theories of intensity (Luce, 2002, 2004).

[3] Consequence monotonicity means that if two gambles differ only in one consequence, the one having the better consequence is preferred. As Luce (2000, p. 45) points out, it implies a form of separability for compound gambles. It also implies backward induction, where each simple gamble in a compound gamble can be replaced by its certainty equivalent. von Winterfeldt, Chung, Luce, and Cho (1997) found few violations of consequence monotonicity for choice-based elicitation procedures, as used in our experiment, and what there was seemed attributable to the variability in certainty equivalence estimates.