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Assessing risky weighting functions for positive and negative binary gambles using the logarithmic derivative function

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HIGHLIGHTS

- Assessing existing proposal for the risky weighting function.
- Assessing the risky weighting function for positive and negative prospects.
- Support of the exponential odds model.

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ABSTRACT

This paper uses the tool of the logarithmic derivative function (LD) to ascertain the functional form of the risky weighting function for probabilistic outcomes. The LD function of a continuous function $g(x)$ is defined as the ratio of the derivative of a function at x to the function itself. The LD is particularly sensitive to changes in the slope of a function thus making it an effective way of distinguishing functions with similar forms. The present study replicates earlier analyses of the LD candidates for the risky weighting function for positive binary gambles and extends the program of study to include negative binary gambles. Empirical estimates for LD values were elicited in an experiment in which participants matched gambles with positive outcomes or with negative outcomes. The risky weighting function for positive and negative gambles differed significantly only in the low range of probability values where $p \leq 0.15$. Several candidate models were shown to be incompatible with the observed LD pattern across both types of gambles. Other candidates had a functional form that was similar to the observed LD pattern, but systematically misfit the observed data in one or more regions of the curve. Of the models that predicted the right shape, only one – the Exponential Odds function $\omega(p) = \exp\left(-s\frac{(1-p)^b}{p^a}\right)$ – showed a random error pattern. The Exponential Odds function was also the only candidate function with best-fitting parameters that differed between positive and negative gambles and thereby picking up on an important difference between the two gamble types.

1. Introduction

Many different economic decisions have costs and are associated with potential consequences. As such, choices with potential gains and losses can be modeled with gambles. If we believe for any reason that the value of a gamble is worth more than its cost, then we are likely to play that gamble. Thus, understanding the perceived worth of a gamble is an important problem for both psychology and economics. Moreover, an individual's subjective weighting of probability is more predictive of choices than the actual probability associated with an outcome (Kahneman & Tversky, 1979). In his famous paradox, Allais (1953) produced a case in which it would be reasonable to make different choices in two situations that were functionally identical. Allais proposed a pair of proposed gambles that differed only in that one gamble had a 1% chance larger chance that the gambler would walk away empty-handed. That difference of 1% looms large when choosing between a certain gain and a 99% chance of gain but relatively small when choosing between a 10% chance of gain and an 11% chance of gain, even when the monetary value associated with each outcome is the same. Allais's thought experiment has since been borne out experimentally (e.g., Chew & Waller, 1986). Thus, it is the extent to which we believe that we can win a gamble and not the actual probability that

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we can win that will determine whether or not we choose to play. Alternatively, we might actually perceive the probability values for a gamble but nonetheless do not weight an outcome simply by the probability for the outcome. In either case it has proven valuable to introduce a functional transform of probability as the weighting function of an outcome utility (Tversky & Kahneman, 1992). The function of objective probability is called here the risky weighting function and is denoted as $\omega(p)$. By using a nonlinear risky weighting function the problem of Allais paradox can be circumvented.

Risky weighting functions assume that an individual will reliably transform different probabilities across the interval from 0 to 1 in consistent ways between choices. One requirement of the functional form of the risky weighting function is that it be able to capture the consistent finding that individuals tend to overweight small probabilities and to underweight large probabilities.[1] If we denote a binary gamble where outcome of V_1 occurs with probability p, otherwise outcome V_2 occurs, as $G(V_1, p, V_2)$, then a generic representation for the gamble utility is given in Eq. (1).

$$U(G) = \omega(p)u(V_1) + [1 - \omega(p)]u(V_2). \tag{1}$$

In essence, (1) is a modification of the expected utility model where the risky weighting function replaces the probability weighting of the outcome utilities, $u(V_1)$ and $u(V_2)$. Please note that this representation does not require any assumptions of specific models of risky decision-making (e.g., Cumulative Prospect Theory as proposed by Tversky & Kahneman, 1992) but merely states that the perceived utility and risk are (a) not necessarily equivalent to their respective subjective value and probability and (b) that the overall utility and risk of a binary gamble are entirely apportioned to the two options. Chechile and Barch (2013) delineated three assumptions about any rational risky weighting function, i.e. (1) $\omega(p) = 0$ if $p = 0$, (2) $\omega(p) = 1$ if $p = 1$, and (3) $\omega'(p) > 0$ for p. They identify irrationalities if these properties of the weighting function are violated.

There are a number of candidate models for the risky weighting function, and since they all are meant to fit similar empirical data, they tend to have similar functional forms. Fig. 1 shows two such candidates: the Prelec (1998) model

$$\omega(p) = e^{-s(-\ln(p))^a} \tag{2}$$

and the Goldstein and Einhorn (1987) model

$$\omega(p) = \frac{sp^a}{sp^a + (1-p)^a}. \tag{3}$$

With the respective parameterizations of the two functions as plotted in Fig. 1, the plots of the function are only perceptibly different in the range of small probabilities – and even there are quite similar – and thus it would seem that both should be satisfactory to model observed patterns of perceived probability.

However, the value of risky weighting functions goes beyond data-fitting. Risky weighting functions are derived based on the assumption of principles of decision-making. For example, the Prelec function is based on an assumption that resolves problems of the class of the Allais functions (the principles behind each of the candidate models will be discussed in Section 2). Thus, if we can discriminate between these similar functions, we can test the relative validity of the underlying principles of decision-making.

Naturally, this discrimination is difficult. It is possible to use statistical model selection techniques to compare model fit. For example, Stott (2006) analyzed 256 combinations of candidate functions (he also included a small class of error terms, which are predicated on the idea that individuals may not always correctly indicate the choices they actually prefer) and evaluated model fit with the Akaike Information Criterion (Akaike, 1973). This approach accounts for a great number of possible functions, but it has its shortcomings: model selection statistics may not agree across models (Myung, 2000) and they assess quantitative but not qualitative aspects of the model fit (Chechile & Barch, 2013). Other investigators have used non-parametric elicitations of risky weighting functions. For example, Abdellaoui (2000) compared prospects with five different probabilities (1/6, 2/6, 3/6, 4/6, and 5/6) to certain outcomes. This method does not stipulate a utility function, providing for direct study of risk perception, but this approach is vulnerable to Allais-types paradoxes (Von Nitzsch & Weber, 1988), as outcomes that are certain tend to elicit choice behavior that is distinct from choices made under risk. Comparison between two risky choices is thus preferable. Bleichrodt and Pinto (2000) used a paradigm that asked participants to compare choices of different probabilities, but tested only five probability values (0.10, 0.25, 0.50, 0.75, and 0.90), limiting the precision of their investigation. A preferable approach would be one that can discriminate among small differences between functions, does not require an assumed utility function, and one that compares judgments made in risky choice to other judgments made in the same domain.

For those reasons, Chechile and Barch (2013) used transformations of candidate models for the risky weighting function that are easier to differentiate, taking the logarithmic derivative $\eta(p)$ of each. The logarithmic derivative of a function is the ratio of the derivative of a function at a given point to the function itself at that point; in the case of a risky weighting function of objective probability p, it is $\eta(p) = \frac{\omega'(p)}{\omega(p)}$. Chechile and Barch (2013) proved a number of general properties about the logarithmic derivative (LD) function of $\omega(p)$. For example, it was shown that (1) the LD function diverges as p approaches 0, (see their Theorem 2), (2) the LD function is positive for all p (see their Theorem 1), (3) $\omega(p) = \exp\left[-\int_p^1 \eta(y)dy\right]$ (see their Theorem 3), and (4) the LD function cannot be monotonically increasing for all probability values (see their Theorem 4).

Because the LD function measures the rate of change of a function relative to the function itself over each point of the domain of the function, it is especially sensitive to small changes in slope, which is precisely what is required to differentiate between functions with similar functional forms. Fig. 2 is a plot of three logarithmic derivative functions of candidate risky weighting functions: clear differences arise after the functions are transformed. Although the risky weighting function for these

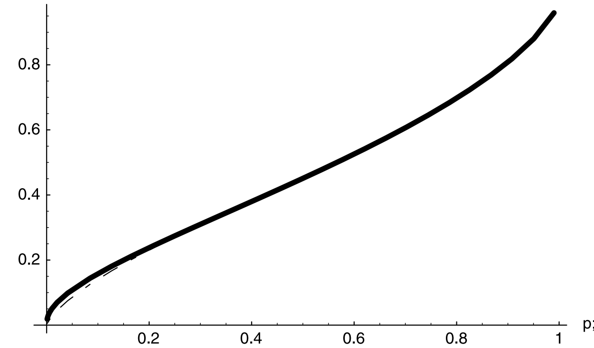

Fig. 1. Similar $\omega(p)$ functions: Prelec (solid; Eq. (2)) with $a = 0.65$ and $s = 1.05$ and Goldstein–Einhorn with $a = 0.68$ and $s = 0.78$ (dashed; Eq. (3)). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

[1] Some candidates for the risky weighting function, for example, the Prelec function (Prelec, 1998) can, given certain parameter values, also describe the opposite pattern were it observed. This pattern was not observed for any of the participants in the current research, and is rarely discussed in the experimental literature.

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