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Advanced analysis of quantum contextuality in a psychophysical double-detection experiment

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HIGHLIGHTS

- Behavioral experiments typically exhibit inconsistent connectedness.
- Most experiments are confined to cyclic systems of binary random variables.
- We present analysis of a non-cyclic psychophysical system.
- We use an advanced version of the Contextuality-by-Default theory to do this.
- The results reveal no contextuality: all of context-dependence consists in inconsistent connectedness.

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ABSTRACT

The results of behavioral experiments typically exhibit inconsistent connectedness, i.e., they violate the condition known as "no-signaling," "no-disturbance," or "marginal selectivity." This prevents one from evaluating these experiments in terms of quantum contextuality if the latter understood traditionally (as, e.g., in the Kochen–Specker theorem or Bell-type inequalities). The Contextuality-by-Default (CbD) theory separates contextuality from inconsistent connectedness. When applied to quantum physical experiments that exhibit inconsistent connectedness (due to context-dependent errors and/or signaling), the CbD computations reveal quantum contextuality in spite of this. When applied to a large body of published behavioral experiments, the CbD computations reveal no quantum contextuality: all context-dependence in these experiments is described by inconsistent connectedness alone. Until recently, however, experimental analysis of contextuality was confined to so-called cyclic systems of binary random variables. Here, we present the results of a psychophysical double-detection experiment that do not form a cyclic system: their analysis requires that we use a recent modification of CbD, one that makes the class of noncontextual systems more restricted. Nevertheless our results once again indicate that when inconsistent connectedness is taken into account, the system exhibits no contextuality.

In recent years there were many reports of behavioral experiments (Accardi, Khrennikov, Ohya, Tanaka, & Yamato, 2016; Aerts & Sozzo, 2014, 2015; Aerts, Sozzo, & Veloz, 2015; Asano, Hashimoto, Khrennikov, Ohya, & Tanaka, 2014; Bruza, Kitto, Ramm, & Sitbon, 2015; Cervantes & Dzhafarov, 2017; Dzhafarov, Zhang, & Kujala, 2015; Khrennikov, 2015; Sozzo, 2015; Wang, Solloway, Shiffrin, & Busemeyer, 2014; Zhang & Dzhafarov, 2017) aimed at (or interpretable as aimed at) revealing *contextuality* of the kind predicted by and experimentally confirmed in quantum physics (Bell, 1964; Clauser, Horne, Shimony, & Holt, 1969; Fine, 1982; Hensen et al., 2015; Klyachko, Can, Biniciopğlu, & Shumovsky, 2008; Kochen & Specker, 1967; Kurzyński, Ramanathan, & Kaszlikowski, 2012; Lapkiewicz et al., 2011). All known to us behavioral data, however, violate a certain condition that makes a direct application of the traditional quantum contextuality analysis impossible. This condition is variously called "no-signaling" or "no-disturbance" in quantum physics (Bacciagaluppi, 2015, 2016; Cereceda, 2000; Kofler & Brukner, 2013; Kurzyński, Cabello, & Kaszlikowski, 2014; Popescu & Rohrlich, 1994; Ramanathan, Soeda, Kurzyński, & Kaszlikowski, 2012) and "marginal selectivity" in psychology (Dzhafarov, 2003; Townsend & Schweickert, 1989; Zhang & Dzhafarov, 2015). It is a required condition for the traditional quantum contextuality analysis, even though it is often violated in quantum mechanical experiments as well (this issue was first systematically discussed

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in Adenier & Khrennikov, 2007; see also Adenier & Khrennikov, 2016; Lapkiewicz et al., 2011, 2013). The Contextuality-by-Default (CbD) theory (de Barros, Dzhafarov, Kujala, & Oas, 2015; Dzhafarov, 2016; Dzhafarov & Kujala, 2014a,b, 2015, 2016a,b, 2017a, in press; Dzhafarov, Kujala, & Cervantes, 2016; Dzhafarov, Kujala, & Larsson, 2015) overcomes this difficulty by proposing a principled way of separating contextuality proper from *inconsistent connectedness* (the CbD term for violations of the "no-signaling" or "marginal selectivity" condition). This theory was used to reanalyze the behavioral experiments aimed at contextuality, with the conclusion that they provide no evidence for contextuality (Cervantes & Dzhafarov, 2017; Dzhafarov, Kujala, Cervantes, Zhang, & Jones, 2016; Dzhafarov, Zhang, & Kujala, 2015; Zhang & Dzhafarov, 2017): inconsistent connectedness is the only form of context-dependence that we have in them. By contrast, when CbD is used to reanalyze a quantum-mechanical experiment that exhibits inconsistent connectedness (Lapkiewicz et al., 2011), contextuality proper (on top of inconsistent connectedness) is established beyond doubt (Kujala, Dzhafarov, & Larsson, 2015).

Virtually all experiments aimed at revealing contextuality, both in quantum physics and in behavioral sciences, deal with a special kind of systems of random variables, called *cyclic systems* in CbD (Kujala et al., 2015). In these systems each property is measured in precisely two different contexts, and each context contains two properties being measured together. If, in addition, all random variables in the system are binary (each indicating presence or absence of a certain property), then the system is amenable to complete and exhaustive contextuality analysis (Dzhafarov & Kujala, 2016a; Dzhafarov, Kujala, & Cervantes, 2016; Dzhafarov, Kujala, & Larsson, 2015; Kujala et al., 2015). In spite of their prominence in quantum theory, however, it is highly desirable to extend contextuality analysis beyond the class of cyclic systems. Many researchers (although not the present authors) find the lack of contextuality in behavioral data to be a disappointing negative result. What if this result is due to the fact that cyclic systems in human behavior are too simple? What if it is "too easy" for a cyclic system to be noncontextual? These are valid questions, and they will have no definite answers until we have a predictive theory of (at least certain types of) human behavior on a par with quantum mechanics.

In the absence of a predictive theory, the only, admittedly imperfect way of dealing with these considerations is to expand the experimentation and contextuality analysis to progressively broader classes of systems. In this paper we make a first step in this direction by analyzing a psychophysical experiment whose results form a non-cyclic system of random variables. This experiment was reported previously (Cervantes & Dzhafarov, 2017), but its analysis was confined to extracting from it a large number of cyclic subsystems and showing all of them to be noncontextual. It is mathematically possible, however, that a system is contextual with all its cyclic subsystems being noncontextual.

A satisfactory way to expand the contextuality analysis beyond cyclic systems was proposed in a recent modification of CbD, dubbed "CbD 2.0" (Dzhafarov & Kujala, 2017a, in press): it is essentially the original CbD in which the measurements of the same property (say, responses to the same stimulus) are analyzed in pairs only. This modification has compelling reasons behind it. The main one is that in the modified theory a subsystem of a noncontextual system is always noncontextual. Another reason is that contextuality analysis is reduced to the problem of compatibility of two *uniquely defined* sets of distributions: the empirically known distributions of context-sharing random variables and the distributions of the "multimaximal couplings" of the random variables measuring the same property in different contexts. All of this is clarified below (Section 2). The modification in question does not affect the theory of cyclic systems, so the results mentioned earlier remain unchanged. However, when it comes to non-cyclic systems, the modification makes the requirements that a system should satisfy to be noncontextual more stringent.

The plan of the paper is as follows. In Sections 1 and 2 we present the basics of the CbD theory, in the "CbD 2.0" version. The discussion is primarily confined to systems of binary random variables (dichotomic measurements), both for simplicity and because the double-detection experiment to be analyzed involves only dichotomic judgments. In Section 3 we apply this theory to the results of our double-detection experiment. Our conclusion is that in spite of the notion of noncontextuality we use being more restrictive than in the original version of the CbD theory, the double detection experiment does not exhibit any contextuality.

1. Introduction to contextuality

Every experiment results in a system of random variables. In most physics experiments these random variables are interpreted as measurements of properties, in most behavioral experiments they are interpreted as responses to stimuli, such as questions. For brevity we will use the term "measurement" in both meanings (because responding to a stimulus can always be viewed as a form of measurement). *What* is being measured therefore is part of the identity of a random variable representing a measurement. It is referred to as the *content* of the random variable. The content, however, does not specify a random variable uniquely, because one and the same content can be measured under different conditions, referred to as *contexts*. For instance, if a content q is measured simultaneously with measurements of other contents, in some cases q' and in other cases q'', then in the former cases the context is $c = (q, q')$ and in the latter ones it is $c' = (q, q'')$. As in Dzhafarov and Kujala (2016a, 2017a), we will write "conteXt" and "conteNt" to prevent their confusion in reading. The conteXt and conteNt of a random variable uniquely identify it within a given system of random variables. So each random variable in a system is double-indexed, R_q^c.

According to the CbD theory's main principle (Dzhafarov, 2016; Dzhafarov & Kujala, 2014a, 2016a,b, in press; Dzhafarov, Kujala, & Cervantes, 2016), two random variables R_q^c and $R_{q'}^{c'}$ are jointly distributed if and only if $c = c'$, i.e., if and only if they are recorded in the same conteXt. Otherwise they are *stochastically unrelated*, i.e., joint probabilities for them are undefined. This means, in particular, that any two R_q^c and $R_q^{c'}$ with the same conteNt in different conteXts are stochastically unrelated (which implies, among other things, that they can never be considered to be one and the same random variable). Their individual distributions may be the same but they need not be. If these distributions are different, the system exhibits a form of context-dependence. However, in CbD, this context-dependence by itself does not say that the system is contextual in the sense related to how this term is used in quantum mechanics. Rather the difference in the distributions is treated as manifestation of information/energy flowing to the measurements of conteNt q from elements of the contexts c, c' other than q. We will refer to this transfer of information/energy as *direct cross-influences*. Thus, if $c = (q, q')$ and $c' = (q, q'')$, the conteNt q does, of course, directly influence its measurement, but, with q fixed, the second conteNt in the pair can also affect this measurement. This can sometimes be attributed to some physical action of q' or q'' upon the process measuring q, or (as another form of information transfer) it can be a form of contextual bias, a change in the procedure by which q is measured

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