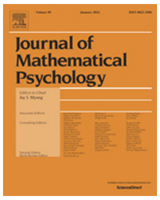
Journal of Mathematical Psychology ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmp



A quantum-like model of selection behavior

Masanari Asano[a], Irina Basieva[b], Andrei Khrennikov[b],*, Masanori Ohya[c], Yoshiharu Tanaka[c]

[a] Liberal Arts Division, National Institute of Technology, Tokuyama College, Gakuendai, Shunan, Yamaguchi 745-8585, Japan
[b] International Center for Mathematical Modeling in Physics and Cognitive Sciences Linnaeus University, Växjö-Kalmar, Sweden
[c] Department of Information Sciences, Tokyo University of Science Yamasaki 2641, Noda-shi, Chiba, 278-8510, Japan

HIGHLIGHTS

- A new model of selection behavior under risk is developed.
- This is a quantum-like model exploring the formalism of quantum probability.
- It explains the famous examples of anomalies for the expected utility theory, Ellsberg paradox, Machina paradox and the disparity between WTA and WTP.

ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Prospect theory
Ellsberg paradox
Machina paradox
WTA and WTP
Quantum-like approach

ABSTRACT

In this paper, we introduce a new model of selection behavior under risk that describes an essential cognitive process for comparing values of objects and making a selection decision. This model is constructed by the quantum-like approach that employs the state representation specific to quantum theory, which has the mathematical framework beyond the classical probability theory. We show that our quantum approach can clearly explain the famous examples of anomalies for the expected utility theory, the Ellsberg paradox, the Machina paradox and the disparity between WTA and WTP. Further, we point out that our model mathematically specifies the characteristics of the probability weighting function and the value function, which are basic concepts in the prospect theory.

1. Introduction

Many studies on selection behavior have been done mainly in economics and psychology. In economics, the *expected utility theory* (Neumann & Morgenstern, 1953) is traditionally used to discuss selection behaviors under risk, which are regarded as *normative* and *rational* from the view of the probability theory. In psychology, anomalies for the expected utility theory have been verified through a large number of experimental tests.

Among the two disciplines, behavioral economics based on the *prospect theory* (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992) has been developed. The prospect theory is categorized into subjective expected utility (SEU) approach that tries to explain the anomalies by simulating the decision maker who makes a choice for maximizing the value of SEU. The SEU in the prospect theory is defined by the *probability weighting function* and the *value function*. The probability weighting function represents the psychological tendency to overestimate small probabilities and underestimate large ones. The value function represents the tendency that a loss gives a greater feeling of pain compared to the joy given by an equivalent gain. (The amount of loss or gain is measured from reference point whose position fluctuates situationally.) However, the development of experimental economics brought the finds of anomalies that cannot be captured in the prospect theory or its gentle modification. For examples, the anomalies shown by Birnbaum (2008), Ert and Erev (2013), Payne (2005) and Thaler and Johnson (1990) are well known and they are difficult to explain.

In recent years, trying to find a theory/model that can explain all anomalies is a major topic in behavioral economics and, many researchers compete for developing descriptive decision-making model.[1]

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* Corresponding author.
E-mail address: Andrei.Khrennikov@lnu.se (A. Khrennikov).

[1] Actually, Ert and Erev (2013) proposed and organized the fair competition of model (*From Anomalies to Forecasts: Choice Prediction Competition for Decisions under Risk and Ambiguity* (CPC2015)). The details of this competition are reported in the website http://departments.agri.huji.ac.il/cpc2015.

http://dx.doi.org/10.1016/j.jmp.2016.07.006
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Please cite this article in press as: Asano, M., et al., A quantum-like model of selection behavior. Journal of Mathematical Psychology (2016), http://dx.doi.org/10.1016/j.jmp.2016.07.006

In this paper, we propose a new decision-making model that is not a gentle modification of SEU. It is a model designed in the *quantum-like approach*, where the state representation specific to the *quantum mechanics* is employed.[2] Quantum mechanics is originally established for the description of statistical properties in microscopic phenomena. The fundamental assertion in the quantum-like approach is that the formalism beyond the probability theory is also applicable to anomalies in various phenomena not limited to the microscopic cases.

We point out that there exist many quantum-like models designed under this assertion (Accardi, Khrennikov, & Ohya, 2008, 2009; Asano, Basieva, Khrennikov, Ohya, & Tanaka, 2012a,b; Asano, Masanori, Tanaka, Khrennikov, & Basieva, 2011b; Asano, Ohya, & Khrennikov, 2011c; Asano, Ohya, Tanaka, Khrennikov, & Basieva, 2011a; Basieva, Khrennikov, Ohya, & Yamato, 2010; Busemeyer & Bruza, 2012; Busemeyer, Matthews, & Wang, 2006b; Busemeyer, Santuy, & Lambert-Mogiliansky, 2008; Busemeyer, Wang, & Lambert-Mogiliansky, 2009; Busemeyer, Wang, & Townsend, 2006a; Cheon & Takahashi, 2006, 2010; Conte et al., 2008, 2009, 2006; De Barros & Suppes, 2009; Dzhafarov & Kujala, 2012; Haven & Khrennikov, 2009, 2013; Khrennikov, 2003, 2004a,b, 2006, 2009a,b, 2011a,b; Khrennikov, Basieva, Dzhafarov, & Busemeyer, 2014; Khrennikov & Haven, 2009c; Ohya & Volovich, 2011; Pothos & Busemeyer, 2009).[3]

Our main interest is not just to make a model that fits experimental data of anomalies, but *to offer the foundation for the new theory of expected theory*, that is, to describe human behavior using non-classical probabilities. We believe that in order to develop the quantum-like approach further to become an established theory, we need to investigate in a deep way how the quantum-like approach compares to prospect theory. In this paper, we will show that the characters of the probability weighting function and the value function can be realized mathematically in a part of our quantum-like model. Further, we will show that our model can explain several anomalies including non-classical ones. These results suggest that our quantum-like model has the potential to be a mile stone toward the development of model to cover all known and yet unknown anomalies.

In Section 2, we mathematically define a cognitive process essential to make a preference. Firstly, the decision maker in our model is aware of the existence of objects to be compared. The structure of awareness is represented by *density operator* (*matrix*), which is the most general state representation in quantum mechanics. We call it the *comparison state*. The decision maker secondly compares the values (utilities) of objects quantitatively. This functionality is represented as a mapping from the comparison state to a real number, which is called the *evaluation function*. A selection decision is derived from the above comparison state and evaluation function. In Section 2.3, we discuss the modeling of selection between two lotteries. The main problem is how to embed the probability distributions into the comparison state. Especially, the comparison state designed in Section 2.3.3 is important, because it is closely related with the crucial concept in quantum theory: The *state transition*, which occurs when a physical value is measured on a system, is the basic assumption in quantum theory. A physical state before measurement is generally represented with the form called *quantum superposition* that is clearly distinguished from a statistical description as obtained after measurement. If the comparison state at the stage before drawing the lots is represented with quantum superposition, the character of the probability weighting function can be explained, see Fig. 1 in Section 2.3.4.

It is also crucial that our model describes the cognitive process that is never explained in the expected utility theory. For example, for the lottery that pays \$100 or \$1 according to a probability distribution, a decision maker might fear to get \$1 by missing the chance of \$100. Then, the difference of these potential outcomes will affect his/her *evaluation of risk*. Such a process is to be experienced before drawing the lottery but never after drawing, and actually, its effect is represented in the comparison state with quantum superposition, as the parameter called *degree of evaluation of risk* (DER). We expect, the evaluation of risk is an essential cause of anomaly. In Section 3, this point will be shown clearly, where we simulate the famous anomalies in selection behavior under *ambiguity*; *Ellsberg paradox* (Ellsberg, 1961) and the *Machina paradox* (Machina, 2009).[4] Ellsberg paradox, see Table 1, is the first example that shows the anomaly due to the *ambiguity aversion*, which lets one to prefer the known risk to the unknown risk. The Machina paradox, see Table 3, points out an existence of anomaly that is impossible to be explained even in the popular models of ambiguity aversion. Our analyses are summarized in the diagrams of Figs. 2 and 3 in Section 3.

In Section 4, we discuss the determination of *cash equivalent* (CE), which is an amount of money whose value is indifferent from a given lot. The CE is related to the *willingness to accept* (WTA) and *willingness to pay* (WTP) (Horowitz & Mcconnell, 2003), each of which is interpreted as cash determined in the aim of selling or buying the lot. It is well-known that the relation of WTA $>$ WTP is generally observed in experimental tests, and this disparity is an important topic in economics. As seen in Fig. 4, CE is defined as the function of the degree of evaluation of risk (DER). Therefore, we can explain the disparity from DER whose value is to be changed depending on the decision maker's situation. Note that in such a situation dependency is consistent with the character of value function in the prospect theory.

2. A model of selection behavior

There are two lots, say A and B. If you chose A, you will get outcome x_i $(i = 1 \dots n)$ *with probability* P_i. *If you chose B, you will get outcome* x_i *with probability* Q_i. *All of the outcomes are different from each other. Which lot do you select?*

When a decision maker decides the preference $A \succ B$ or $B \succ A$ in this situation, he/she will recognize the following three points.

1. *What* objects exist,
2. *Which* pairs of objects are to be compared,
3. *How* comparisons are evaluated.

Here, we have to emphasize that "objects" mean "events" that will be experienced in the future. He/she can simulate the experience that he draws the lot A (B) and gets the outcome x_i. Let us represent such an event by (A, x_i) or (B, x_i). Further, we assume that the decision maker sets the utilities of (A, x_i) and (B, x_i) by $u(x_i) \equiv u_i$. (The utility of event depends on only outcome.) Here, $u(x)$ is a utility function of outcome x. Under this assumption, comparing

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[2] The problem of the state interpretation is one of the most complicated problems of quantum foundations. The present situation is characterized by a huge diversity of interpretations (Khrennikov, 2009a). One of them is *the information interpretation* which was strongly supported by the recent quantum information revolution. We shall use this interpretation. Thus a quantum-like mental state represents information available to decision makers and structured in the special (quantum-like) way by their brains.

[3] The quantum-like approach works very well to model a variety of behavioral effects, e.g., the order effect or disjunction and conjunction effects. However, it is not clear whether the mathematical formalism of quantum theory can serve to model all possible behavioral phenomena, see Boyer-Kassem, Duchene, and Guerci (2016), Boyer-Kassem, Duchene, and Guerci (in press) and Khrennikov et al. (2014) for the recent analysis of this problem.

[4] There exists another quantum-like approach trying to solve the two paradoxes, see Aerts, Sozzo, and Tapia (2012), in which the effect of DER is not assumed.

Please cite this article in press as: Asano, M., et al., A quantum-like model of selection behavior. Journal of Mathematical Psychology (2016), http://dx.doi.org/10.1016/j.jmp.2016.07.006