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Quantum cognition and Bell's inequality: A model for probabilistic judgment bias

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HIGHLIGHTS

- A quantum model predicting judgments in a Wigner–d'Espagnat-based tasks.
- The model predicts either incompatibility between variables or entanglement.
- Violations of the inequality in line with the model's predictions.
- Violations may be attributed to an explicit subadditivity pattern.

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ABSTRACT

Recently, quantum theory has shown its effectiveness in modeling psychological phenomena. Given the importance of Bell's inequality in the context of quantum physics, this work aims to investigate this issue in the domain of human probabilistic reasoning. Here, we present two quantum models that are able to predict the employment of the representativeness heuristic in a probabilistic task based on Bell's inequality in the Wigner–d'Espagnat format. The difference between the two models is based on the origins of the correlations achievable in conceptual combination; the first assumes incompatible variables while the second is based on quantum entanglement. From these models, two different scenarios related to three dichotomous variables $(A, \bar{A}), (B, \bar{B}), (C, \bar{C})$ were created. Each scenario was manipulated in order to predict the violation of the inequality $(Pr(A \cap \bar{C}) > Pr\left((A \cap \bar{B}) \cup (B \cap \bar{C})\right))$ or not $(Pr(A \cap \bar{C}) \leq Pr\left((A \cap \bar{B}) \cup (B \cap \bar{C})\right))$. Each condition was tested using two different modalities of response: Forced choice and probability rating of a single sentence. In Experiment 1, participants were randomly assigned to a single scenario, condition, and modality of response. The data showed a violation of the inequality consistent with the predictions of both models. In Experiment 2, we investigated the influence of an explicit subadditivity pattern (i.e., if $Pr\left((A \cap \bar{B}) \cup (B \cap \bar{C})\right) \leq Pr(A \cap \bar{B}) + Pr(B \cap \bar{C})$) in our tasks, both from an empirical and theoretical point of view. Our results confirm the use of the quantum cognition approach in developing cognitive models.

1. Introduction

A long-standing tradition of psychological research has found a substantial disagreement between classic (Bayesian) probability theory and human judgments (Kahneman & Frederick, 2005; Kahneman & Tversky, 2000; Tversky & Kahneman, 1974). Those observations, together with the strong order and context dependency of human decision making (Hogarth & Einhorn, 1992; Schwarz & Sudman, 2012), have induced several researchers to propose that quantum probability theory could provide a better representation of human thinking when compared to classical models (Busemeyer & Bruza, 2012; Pothos & Busemeyer, 2013; Wang, Busemeyer, Atmanspacher, & Pothos, 2013). Indeed, over the last decade the quantum cognition approach has shown its effectiveness in modeling psychological phenomena; for example, in explaining classical human probability judgment errors, such as conjunction and disjunction errors (Busemeyer, Pothos, Franco, & Trueblood, 2011).

In the development of quantum theory, Bell's theorem has played a fundamental role (Wiseman, 2006). Using his famous inequalities, Bell demonstrated that quantum models can account

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for simultaneous correlations between variables that are stronger than any classical prediction. Here, we take advantage of such strong quantum correlations to model the probabilistic reasoning biases observable in human judgments. In the following, we explore the issue of Bell's inequality violation in the context of human probabilistic reasoning. We developed two quantum models that are able to predict probabilistic judgments and the employment of representativeness heuristic in a Bell's inequality-based task. In particular, we employed the Wigner–d'Espagnat version of Bell's inequality (d'Espagnat, 1979; Harrison, 1982; Stapp, 1979; Wigner, 1970) that is easily applicable in the domain of probabilistic judgments.

2. Bell's theorem in physics and in psychology

2.1. Bell's inequality

In 1964, Bell published his famous theorem (Bell, 1964). According to Bell's theorem, "no physical theory of local hidden variables can ever reproduce all of the predictions of quantum mechanics" (Parker, 1994, p. 542).

From an historical point of view, in the 1930s there was an intense philosophical debate over the implications of the Copenhagen interpretation (Heisenberg, 1930) of quantum theory. Within this debate, Albert Einstein, Boris Podolsky and Nathan Rosen proposed that quantum theory was incomplete under the assumptions of locality and realism (Einstein, Podolsky, & Rosen, 1935). In particular, they referred to quantum entanglement, the phenomenon in which pairs of particles must be described as a whole because they interact in such a way that the state of each particle cannot be described independent of the other. Interestingly, since the 1920s, it has become apparent that a single particle of an entangled pair possesses knowledge of what kind of measurement (and what outcome) has been taken out on the other particle even if there is no apparent means for such information to be communicated between them (independent of time and spatial constraints). However, Einstein, Podolsky and Rosen argued that an "element of reality" that is currently unmeasurable (a hidden variable) can account for the entanglement phenomenon. Thus, assuming locality (i.e., an object is influenced directly only by its immediate surroundings) and realism (i.e., what exists in the physical world is logically and conceptually independent by measurement),[1] quantum theory is incomplete because it does not provide a complete description of the system (Einstein et al., 1935).

In 1964, starting from the same two assumptions held by Einstein et al. (1935) (i.e., locality and realism), Bell derived his famous inequality. He demonstrated that the violation of such inequality entails that at least one of the two assumptions must be false (Bell, 1964). This result has stimulated a lot of theoretical (and empirical) research. As a matter of fact, as inspired by the original paper, there are now many different versions of Bell's inequality. Moreover, in the last 50 years, the complexity of the debate has increased along with many divergences in the interpretation of the problem's terms.

The original Bell's inequality concerns experiments conducted on pairs of particles that after a strong interaction (entanglement) have been physically separated. The typical example is based on the simultaneous measurement of the spin vector of the two particles that can only have two distinct values per orientation of the spin. Consider measuring three such two-valued properties: A, B, and C; there could be three different angles of the spin vector. Under the assumption of locality and realism, it is possible to demonstrate that for a classical system the following inequality always holds: $Pr_{same}(A \cap B) + Pr_{same}(A \cap C) + Pr_{same}(B \cap C) \geq 1$, while for a quantum system, it is possible to have configurations of the two particles, states for which the inequality is violated.

A key aspect that needs to be stressed is the simultaneity of the measurement on the two particles. Indeed, if the two measurements are sequential and between the first and the second measurement the two systems are able to communicate (signaling), a classical model can predict the violation of Bell-type inequalities.[2] The no-signaling condition (i.e., the impossibility of instantaneous messaging at a distance) is then essential to demonstrate, via the violation of Bell's inequality, entanglement between particles and the non-locality of quantum mechanics.

The original inequality also required the perfect anti-correlation of outcomes. Due to the difficulty of having perfect anti-correlation in an actual experiment, Clauser, Horne, Shimony, and Holt (1969) generalized Bell's results by introducing the CHSH inequality (which does not assume perfect anti-correlations). In subsequent decades, other inequalities related to Bell's theorem have been proposed by d'Espagnat (1979), Leggett and Garg (1985), Stapp (1979), and Wigner (1970) among others. Within physics, despite a continuing debate, violations of Bell's inequality have commonly been considered the ultimate proof of entanglement, and more generally, quantum theory. The violations of such inequalities were first convincingly observed in the experiment performed by Aspect, Dalibard, and Roger (1982). Since then, other experiments have confirmed this result, thus supporting quantum theory's predictions (Brida, Degiovanni, Genovese, Schettini, Polyakov, & Migdall, 2008; Franson, 1989; Genovese, 2005; Ou & Mandel, 1988).

2.2. Quantum cognition and Bell's inequalities

Violations of Bell's inequalities determine the necessity to appeal to quantum theory in order to formalize psychological phenomena. As far as we know, the first empirical tests of Bell's inequalities within psychology were performed by Conte, Khrennikov, Todarello, De Robertis, Federici and Zbilut (2008). The authors developed a Bell's inequality-based test employing ambiguous figures but they did not find an actual violation of the inequality. However, this pioneering work provided evidence of a strong link between quantum theory and human cognition. Afterwards, Asano, Khrennikov, Ohya, Tanaka, and Yamato (2014) and Aerts and Sozzo (2014) empirically proved Bell's inequality violations in ambiguous figures and concepts combinations, respectively. For example, starting from a quantum mechanics-based theory for modeling and representing concepts combinations (Aerts & Gabora, 2005a,b), Aerts and Sozzo (2014) investigated how the context influences the typicality of a single exemplar and the applicability of a single property of a concept (the so-called Guppy Effect[3]). They hypothesized that entanglement occurs naturally when two or more concepts are combined in the cognitive system. Constructing some Bell's inequalities based on conceptual combination, the authors empirically showed the violations of such inequalities. Aerts and Sozzo (2014) concluded that formal analysis of how concepts are organized in the cognitive system should take into account the possibility that concepts

[1] These two principles are often referred as a single principle called local realism.

[2] We thank an anonymous reviewer for suggesting to emphasize this point throughout the entire paper.

[3] The "Guppy Effect" refers to the typicality of a conjunctive concept being greater than that of either of its constituents. In the example, a guppy is considered to be a prototypical example of a pet fish, but it is commonly rated far less prototypical for the classes of either pet or fish. Such effect represents a problem for prototype theory (Osherson & Smith, 1981) and for cognitive modeling efforts based on fuzzy set theory.