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Modeling behavior of decision makers with the aid of algebra of qubit creation–annihilation operators

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HIGHLIGHTS

- A general model of decision-making that builds upon the algebra of creation–annihilation operators from quantum information is considered.
- The algebra is used to build phenomenological master equations for dynamics of states of decision makers.
- The model is exemplified with a case of cooperation/non-cooperation of political parties.

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ABSTRACT

We present a general model of the process of decision making based on the representation of the basic behavioral variables with the aid of an algebra of qubit creation–annihilation operators, adopted from the quantum information theory. In contrast to the genuine quantum physical systems, which are divided into either bosons or fermions and modeled with the aid of operators, satisfying canonical commutation or anti-commutation relations, decision makers preferences for possible actions are constructed with the aid of operators satisfying the so-called qubit commutation relations. Systems described by operators, satisfying such commutation relations, combine the features of bosons and fermions. Thus, one of the basic consequences of the presented model is that decision makers mimic the combined bosonic–fermionic behavior. By using the algebra of qubit creation–annihilation operators, we proceed with the construction of the concrete operators, describing the process of decision making. In particular, the generators of the quantum Markov dynamics, which is used for modeling human decision making process, are expressed as polynomials of the qubit creation–annihilation operators. The devised coefficients have a natural cognitive and social meaning.

1. Introduction

During the last two decades, the formalism of quantum mechanics was actively pursued, to model the process of decision making in cognitive psychology, sociology, economics, finance and politics, see, e.g., Accardi, Khrennikov, and Ohya (2008, 2009), Aerts, Sozzo, and Tapia (2012), Asano, Ohya, and Khrennikov (2011a); Asano, Ohya, Tanaka, Basieva, and Khrennikov (2011b, 2012), Bagarello (2012, 2015b), Bagarello and Haven (2016), Basieva, Khrennikov, Ohya, and Yamato (2011), Busemeyer and Bruza (2012), Busemeyer, Wang, and Townsend (2006), Khrennikova (2014a,b, 2015, 2016), Khrennikova and Haven (2016), Khrennikova, Haven, and Khrennikov (2014) and Pothos and Busemeyer (2009).[1] One of the problems of this approach is the absence of an analog of the procedure of canonical quantization, which is used in physics to transfer classical physical quantities defined as functions on the phase space, $f = f(q, p)$, into the corresponding operators acting in complex Hilbert space of states of quantum systems (Schrödinger quantization procedure: $\hat{f} = f(\hat{q}, \hat{p})$). Roughly speaking, we do not have a kind of classical mechanics on the phase space for mental variables. Up to now, we were not able to identify the mental analogues of the position and momentum variables (q, p) and to construct a type of a "mental phase space". One cannot exclude the possibility that such observables would not exist at all. Their existence in physics is closely related to the real manifold geometry of physical space used in classical physics. In principle, there are no reasons to expect that the "mental space" has the same geometry. As a consequence,

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[1] At the same time, see Boyer-Kassem, Duchene, and Guerci (2016, forthcoming), Plotnitsky (2014) for a critical analysis of the ability of quantum formalism to cover all problems arising in mathematical modeling of human reasoning and decision making.

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we are neither able to construct the "quantum phase space" for cognition with the "coordinates" $(\hat{q}, \hat{p})$.

Typically, in quantum models applied to human reasoning and decision making the operators expressing mental entities are developed phenomenologically (Busemeyer & Bruza, 2012; Busemeyer et al., 2006; Pothos & Busemeyer, 2009) by using a heuristic reasoning, e.g., with the aid of the elements of a payoff matrix, e.g., in games of the Prisoner's Dilemma type (Pothos & Busemeyer, 2009). Although such strategy is quite successful, it would be useful to develop a general quantization formalism applicable to the process of decision making. We remark that in quantum physics, besides the Schrödinger quantization procedure, there exists another actively used quantization procedure based on the operators of creation–annihilation $a^{\star}, a$ that is typically explored in quantum fields theory.[2] It is natural to apply this procedure in the quantum-like framework.

The main obstacle preventing a straightforward application of the quantum formalism of the creation–annihilation operators is due to the behavior of genuine quantum physical systems being constrained. These systems always belong to one of the two disjoint classes, namely, bosons or fermions, see Appendix. This separation induces commutation and anticommutation relations, respectively (a detailed synthesis is provided in the Appendix). These standard operator algebras do not correspond to the features of the process of human decision making. At the same time, the quantum-like modeling of decision making matches the standard quantum information representation. As is well known (but not so much emphasized in the quantum information theory), the qubit representation is neither bosonic nor fermionic. In fact, there is a gap between the qubit representation of quantum computing and, for example, its real physical fermionic realization. To transfer the qubit representation into the fermionic one (e.g., for quantum computations with electrons), special mathematical transformations are needed (Bravyi & Kitaev, 2002). On one hand, the recognition that the behavior of a decision maker is neither bosonic nor fermionic simplifies the application of quantum information theory, since the corresponding model construct can be directly nested in a qubit space.

On the other hand, the qubit formalism of creation–annihilation operators is not so widely applied.[3] We can only mention a detailed presentation of this formalism by Frydryszak (2011). One of the primary aims of this paper is to present essentials of this quantization formalism to readers interested in applications of the quantum methods to cognitive psychology and decision theory in sociology, economics and finance. In these interdisciplinary social science applications (by the aforementioned reasons) this formalism is even of a greater importance than in the applications of quantum information theory to physics phenomena, where ultimately one is constrained to operate either with bosonic or fermionic operators.

By using the qubit creation–annihilation formalism we can proceed towards constructions of the concrete operators, describing the process of decision making, in particular, the generators of the quantum-like Markov dynamics, which is used for modeling agents' choice formation. In this modeling we apply *theory of open quantum systems* and the process of approaching final choices is mathematically represented as a Markov approximation of the dynamics of the (mental) state of a cognitive system (a decision maker or a social entity) interacting with some outside environment. The latter is treated from the purely informational viewpoint.[4] The formalism adopted from the theory of open quantum systems has already been successfully approbated on a variety of decision making problems (Accardi et al., 2008, 2009; Asano et al., 2011a,b, 2012; Basieva et al., 2011; Khrennikova, 2014a, 2015, 2016; Khrennikova & Haven, 2016; Khrennikova et al., 2014), by modeling the decision making of players in games of the Prisoner's Dilemma type, models of gene expression and epigenetic evolution, political studies (formalizing voters' behavior in elections and an establishment of cooperation between political parties). However, as was already brought up, the generator-operators of the quantum adaptive dynamics representing the mental state evolution in the process of decision making were selected phenomenologically. In the current contribution we present the general canonical scheme for their construction based on the qubit algebras of creation and annihilation operators.

Examples of possible applications of the algebras of qubit creation and annihilation operators are presented in Section 2 are broad: modeling of actions of states at the world's political arena, cooperation between different political parties at a state's political arena, trader decision making in the process of selling and buying commodities and financial assets, overall decision making by individuals (related to choosing e.g. an accommodation). This paper is conceived to be of a conceptual nature, where the main aim is to theoretically rationalize the usage of the qubit operator-algebras and exemplify the areas of their possible application.

We point once again to the important interpretational consequence of this study. The models of decision making which have been applied outside of physics are operational constructs (the so called "quantum like models") and not genuine quantum physical models. The latter are constrained by clustering all the quantum systems into two disjoint classes, namely bosons (e.g., photons) and fermions (e.g., electrons). The real behavior of microscopic systems is mathematically modeled with the aid of two special operator-algebras, based on canonical commutation and anticommutation relations, respectively. The decision making processes and their features are mathematically well represented by the means of the algebra based of special qubit commutation relations,[5] which are neither bosonic nor fermionic. In particular, this feature distinguishes the quantum-like models of cognition (that adopt the mathematical structure of quantum physics phenomenologically) from the genuine quantum physical models of brain's functioning, cf. Hameroff and Penrose (2014). Such quantum physical models are still based on bosons and fermions.

Finally, we outline the possible generalizations of our formalism. As we already emphasized, in the real nature particles appear

[2] To be consistent with the above notations for the position and momentum operators, we should proceed with the symbols $\hat{a}^{\star}, \hat{a}$, where in the quantization formalism the hats symbolize the operator nature of quantities. However, to simplify notation in long expressions for operators which will be constructed as polynomials of the creation–annihilation operators, we shall skip the hats.

[3] In theoretical quantum computing researchers operate with unitary gates and in real physical applications they have to move either to bosonic or fermionic algebras.

[4] For example, for decision making in finance such an environment contains the information on the real state of economics, world-wide political news, as well as psychological factors, such as expectations of investors related to future price formation on the finance market. In the context of decision making by voters, an election environment contains information related to the economic and finance conditions, political news, but also a variety of psychological biases conveyed by the mass-media during the election campaign (Khrennikova, 2014a, 2015, 2016; Khrennikova & Haven, 2016; Khrennikova et al., 2014).

[5] Of course, this is a statement about the general state of affairs. One cannot exclude a possibility that in some decision making contexts agents' behavior might be in accord with the purely bosonic or fermionic statistics. Finding such empirical examples, e.g., in cognitive psychology, economics, game theory would be of a vast interest. We remark that fermionic creation–annihilation operators were applied by Bagarello (2012, 2015b) and Bagarello and Haven (2016) to model creation of alliances between political parties and the dynamics of buying and selling of financial assets. We also point to exploring of the Fock space formalism for modeling of cognitive phenomena by Sozzo (2014). A more detailed description of the mathematics and social meaning of fermionic and bosonic operators can be found in the Appendix.

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