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Exploring the relations between Quantum-Like Bayesian Networks and decision-making tasks with regard to face stimuli

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HIGHLIGHTS

- Representation of objects in an arbitrary n-dimensional vector space.
- Computation of quantum interference effects through vector similarity functions.
- Usage of contents of images to compute quantum interference parameters.
- Application of a Quantum-Like Bayesian Network to make predictions.

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ABSTRACT

In this work, we propose to model the Categorization/Decision experiment from Busemeyer et al. (2009) with a Quantum-Like Bayesian Network. We also propose the representation of objects (or events) in an arbitrary n-dimensional vector space, enabling their comparison through similarity functions. The computed similarity value is used to set the quantum parameters in the Quantum-Like Bayesian Network model. Just like in the work of Pothos et al. (2013), we are not restricting our model to a vector in a two-dimensional space, but to an arbitrary multidimensional space.

In the end, we conclude that the vector representation of the contents of the images can explain the paradoxical findings and the violations of the laws of classical probability that were found in some works of the literature, suggesting that the contents of the images can already produce some quantum effects.

1. Introduction

The purpose of this work is to explore the applications of the formalisms of quantum mechanics to areas outside of physics, more specifically in domains regarding decision making and cognition.

Quantum cognition has emerged as a research field that aims to build cognitive models using the mathematical principles of quantum mechanics. In this sense, psychological (and cognitive) models benefit from the usage of quantum probability principles because they have many advantages over classical counterparts (Busemeyer, Wang, & Shiffrin, 2015). In quantum theory, events are represented as multidimensional vectors in a Hilbert space. This vector representation comprises potentially for the occurrence of all events at the same time. In quantum mechanics, this property refers to the superposition principle. Under a psychological point of view, a quantum superposition can be related to the feeling of confusion, uncertainty or ambiguity (Busemeyer & Bruza, 2012). This vector representation neither obeys the distributive axiom of Boolean logic nor the law of total probability. It also enables the construction of more general models that can mathematically explain cognitive phenomena such as violations of the Sure Thing Principle (Khrennikov & Haven, 2009; Martínez-Martínez & Sánchez-Burillo, 2016), which is the focus of this study. Quantum probability principles have also been successfully applied in many different fields of the literature, namely in biology (Asano et al., 2012; Asano, Khrennikov, & Ohya, 2015), economics (Haven & Khrennikov, 2013; Khrennikov, 2009), perception (Conte, 2008; Conte et al., 2007), jury duty (Trueblood & Busemeyer, 2011), game theory (Brandenburger, 2010; Mura, 2005), order effects (Wang, Solloway, Shiffrin, & Busemeyer, 2014), opinion polls (Khrennikov & Basieva, 2014; Khrennikov, Basieva, Dzhafarov, & Busemeyer, 2014), etc.

Previously in the literature, Busemeyer, Wang, and Lambert-Mogiliansky (2009) studied the differences between a classical Markov and a quantum dynamical model in order to explain some violations of the law of classical probability theory in a

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categorization experiment. Participants were presented with a set of digitally modified images of faces. Then, they had to first to categorize the face as *Good* or *Bad* and then perform the decision to either *Withdraw* or *Attack*. In the end, the proposed quantum dynamical model was able to accommodate the violations of the laws of classical probability theory by fitting the quantum parameters. This work demonstrated that quantum theory could be applied to build more general models to explain paradoxical situations found in cognitive psychology. More recently, more experiments to investigate the impact of quantum interference effects under the categorization experiment have been performed in the work of Wang and Busemeyer (2016).

In the present work, we propose an alternative way to accommodate the paradoxical findings detected in the experiments of Busemeyer et al. (2009) and Townsend, Silva, Spencer-Smith, and Wenger (2000) that takes only into account the contents of the images and their vector similarities. The current work makes use of a Quantum-Like Bayesian model, initially introduced in the work of Moreira and Wichert (2014), and later developed in the works of Moreira and Wichert (2015a,b, 2016). The similarity is used to fit quantum interference parameters in the Quantum-Like Bayesian Network model. The main advantage of the proposed Quantum-Like Bayesian Network towards other cognitive models is its predictive nature and its scalability. By scalability we mean that the network structure of the proposed model is able to model more complex decision scenarios (scenarios that are modelled with several random variables). Moreover, through the representation of objects (or events) by their contents, one is able to perform vector similarities in an n-dimensional vector space and compute quantum parameters.

Approaching this categorization/decision experiment under a quantum probabilistic point of view is also important for several reasons (Pothos & Busemeyer, 2013). For instance, in the work of Pothos and Busemeyer (2009), the authors showed that a classical Markov model could not explain the violations to the Sure Thing Principle found in the experiment. Of course, one could always model a Markov model with extra hidden states and parameterizations to model these violations. However, this would lead to an exponential increase in complexity. Quantum probability theory is important for this reason. The geometric representation of events, which is present in quantum probability, does not exist in a classical setting. The main advantage of this geometrical representation is the ability of allowing the rotation from one basis into another in order to contextualize events and interpret events, providing great flexibility to decision-making systems.

2. Overview of probabilistic graphical models

In this section, we introduce the concepts of classical and Quantum-Like Bayesian Networks.

2.1. Classical Bayesian Networks

A classical Bayesian Network can be defined by a directed acyclic graph structure in which each node represents a different random variable from a specific domain and each edge represents a direct influence from the source node to the target node. The graph can represent independence relationships between variables, and each node is associated with a conditional probability table that specifies a distribution over the values of a node given each possible joint assignment of values of its parents (Koller & Friedman, 2009).

The full joint distribution (Russel & Norvig, 2010) of a Bayesian Network, where X is the list of variables, is given by:

$$Pr(X_1, \ldots, X_n) = \prod_{i=1}^{n} Pr(X_i | Parents(X_i)). \tag{1}$$

The formula for computing classical exact inferences on Bayesian Networks is based on the full joint distribution (Eq. (1)). Let e be the list of observed variables and let Y be the remaining unobserved variables in the network. For some query X, the inference is given by:

$$Pr(X|e) = \alpha Pr(X, e) = \alpha \left[\sum_{y \in Y} Pr(X, e, y) \right] \tag{2}$$

where $\alpha = \dfrac{1}{\sum_{x \in X} Pr(X = x, e)}$.

The summation is over all possible y, i.e., all possible combinations of values of the unobserved variables y. The α parameter corresponds to the normalization factor for the distribution $Pr(X|e)$ (Russel & Norvig, 2010). This normalization factor comes from some assumptions that are made in Bayes rule.

2.2. Quantum-Like Bayesian Networks

A more recent work from Moreira and Wichert (2014) suggested defining the Quantum-Like Bayesian Network in the same manner as in the work of Tucci (1995), replacing real probability numbers by quantum probability amplitudes.

In this sense, the quantum counterpart of the full joint probability distribution corresponds to the application of Born's rule to Eq. (1). An interesting discussion about the foundations of Born's rule can be found in the article of Deutsch (1988).

$$Pr(X_1, \ldots, X_n) = \left| \prod_{i=1}^{N} \psi_{(X_i | Parents(X_i))} \right|^2. \tag{3}$$

The general idea of a Quantum-Like Bayesian network is that, when performing probabilistic inference, the probability amplitude of each assignment of the network is propagated and influences the probabilities of the remaining nodes. In other words, *every* assignment of *every* node of the network is propagated until the node representing the query variable is reached. Note that, by taking multiple assignments and paths at the same time, these trails influence each other and produce interference effects.

The quantum counterpart of the Bayesian exact inference formula corresponds to the application of Born's rule to Eq. (2), leading to:

$$Pr(X|e) = \alpha \left| \sum_{y} \prod_{x=1}^{N} \psi_{(X_x | Parents(X_x), e, y)} \right|^2. \tag{4}$$

Expanding Eq. (4), it will lead to the quantum interference formula:

$$Pr(X|e) = \alpha \left(\sum_{i=1}^{|Y|} \left| \prod_{x}^{N} \psi_{(X_x | Parents(X_x), e, y=i)} \right|^2 + 2 \cdot Interference \right)$$

$$Interference = \sum_{i=1}^{|Y|-1} \sum_{j=i+1}^{|Y|} \left| \prod_{x}^{N} \psi_{(X_x | Parents(X_x), e, y=i)} \right| \cdot \left| \prod_{x}^{N} \psi_{(X_x | Parents(X_x), e, y=j)} \right| \cdot \cos(\theta_i - \theta_j). \tag{5}$$

In the end, we need to normalize the final scores that are computed to achieve a probability value, because we do not have the constraints of double stochasticity operators. In classical Bayesian inference, normalization of the inference scores is also necessary due to assumptions made in Bayes rule. The normalization factor corresponds to α in Eq. (5).