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A quantum-like model for complementarity of preferences and beliefs in dilemma games

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HIGHLIGHTS

- We take a new look at data collected in a sequential prisoner's dilemma.
- We observe three behavioral effects and argue that two have a quantum-like nature.
- Preferences and beliefs of the players exhibit complementarity.
- We build a model in line with the theory of positive-operator valued measure.
- The model is successfully applied to explain the experimental results.

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ABSTRACT

We propose a formal model to explain the mutual influence between observed behavior and subjects' elicited beliefs in an experimental sequential prisoner's dilemma. Three channels of interaction can be identified in the data set and we argue that two of these effects have a non-classical nature as shown, for example, by a violation of the sure thing principle. Our model explains the three effects by assuming preferences and beliefs in the game to be complementary. We employ non-orthogonal subspaces of beliefs in line with the literature on positive-operator valued measure. Statistical fit of the model reveals successful predictions.

1. Introduction

During the recent decade, there is an increasing interest in decision-making and cognitive models that employ a quantum probabilistic (*QP*) framework. In fact, the application of quantum-like concepts to portray human information processing was considered since the early development of quantum mechanics. For example, Bohr (1950) defended the idea that some aspects of quantum theory could provide an understanding of cognitive processes but never provided a formal cognitive model in light of a QP hypothesis. The so called quantum cognitive theories have only begun to emerge as of late (Busemeyer & Bruza, 2012; Deutsch, 1999; Haven & Khrennikov, 2013; Khrennikov, 2010; Pothos & Busemeyer, 2013; Wang, Solloway, Shiffrin, & Busemeyer, 2014; Yearsley & Pothos, 2014).

QP is defined as the set of mathematical rules used to assign probabilities to events from quantum mechanics (Hughes, 1989; Isham, 1989), but without any of the physics. As it is derived from different sets of axioms than classical probability theory, it is subject to alternative constraints and has the potential to be relevant in any area of science where a need to formalize uncertainty arises. Since encoding uncertainty is a major aspect of cognitive functions in psychology, QP shows potential for cognitive modeling. These studies are not about the use of quantum physics in brain physiology, which is a disputable issue (Hameroff, 2007; Litt, Eliasmith, Kroon, Weinstein, & Thagard, 2006) about which we are skeptical. Rather, we are interested in QP theory as a mathematical framework for cognitive modeling.

Applications of QP theory have been presented in decision-making (Bordley, 1998; Busemeyer, Pothos, Franco, & Trueblood, 2011; Busemeyer, Wang, & Townsend, 2006; Lambert-Mogiliansky, Zamir, & Zwirn, 2009; Pothos & Busemeyer, 2009;

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Trueblood & Busemeyer, 2011; White, Pothos, & Busemeyer, 2014; Yukalov & Sornette, 2011), conceptual combination (Aerts, 2009; Aerts & Gabora, 2005; Blutner et al., 2008), memory (Bruza, 2010; Bruza, Kitto, Nelson, & McEvoy, 2009), and perception (Atmanspacher, Filk, & Römer, 2004). For a detailed study on the potential use of quantum modeling in cognition, see Busemeyer and Bruza (2012) and Pothos and Busemeyer (2013). The majority of models presented in the quantum cognition literature addresses standard aspects of decision-making processes: similarity judgments (Barque-Duran et al., 2016; Pothos et al., 2015; Yearsley, Pothos, Hampton, & Barque-Duran, 2014), the constructive role of articulating impressions (White, Barque-Duran, & Pothos, 2015; White et al., 2014), and order effects in belief updating (Trueblood & Busemeyer, 2011) among numerous other applications.

Little literature has focused on strategic decision-making or game theory. Whenever two or more agents interact, one agent is not only reacting to the information that he receives, but is likewise generating information towards other players. These strategic environments are unique in relation to standard decision-making scenarios under uncertainty, since every agent needs to reason on two parts of the problem: his own actions and his expectations on the opponent's actions. Few studies applying QP instruments to model the way agents process the information in a game have been published with regards to this particular matter: Busemeyer and Pothos (2012), Martínez-Martínez and Sánchez-Burillo (2016), Pothos and Busemeyer (2009), and Pothos, Perry, Corr, Matthew, and Busemeyer (2011). Other approaches in which the quantumness enters through an extension of the classical space of strategies and/or signals have also been discussed, e.g., by Brandenburger (2005), Brunner and Linden (2013), and La Mura (2005); as well as a model to analyze games with agents exhibiting contextual preferences (Lambert-Mogiliansky & Martínez-Martínez, 2015).

In this paper, we describe the application of QP theory to modeling the mutual influence between preferences and beliefs in sequential social dilemmas. This idea was first explored in Martínez-Martínez, Denolf, and Barque-Duran (2015). We present a quantum-like model for preferences and beliefs (QP&B) that replicates the experimental results from Blanco, Engelmann, Koch, and Normann (2014) while providing a novel theoretical approach on cognitive dynamics in strategic interactions. Our model asserts that the relationship between a player's beliefs and his preferences is inherently non-classical and continues the work done in Pothos and Busemeyer (2009) exploiting the ideas of measurement utilized in quantum theory. We redefine these two properties as complementary. In that capacity, they cannot be measured at the same time, as the act of measuring one property alters the state of the other property. The non-classical nature of such a relationship and its application in cognition has already been discussed in, e.g., Denolf and Lambert-Mogiliansky (2016).

2. Experimental design

The data set that our QP&B model deals with is provided by Blanco et al. (2014). Their experiment was designed for explicitly testing different channels through which preferences and beliefs of an agent immersed in a social dilemma may influence each other. As the authors motivate, this experimental evidence is novel and its main interest stems from the fact that previous analyses of strategic interactions considered preferences and beliefs to be independent. This fact implies that the choice of actions in environments with uncertainty can be rationalized as just a best-response to some particular form of belief about the possible states of the world or about the action that is expected to be played by an opponent.

2.1. Standard version of the prisoner's dilemma game

The symmetric prisoner's dilemma game is a game involving two players, player I and player II, who can choose among two actions: cooperate (C) or defect (D). The normal form of this game is defined by the following 2×2 payoff matrix

		Player II	
		C	D
Player I	C	(π_c, π_c)	(π_b, π_a)
	D	(π_a, π_b)	(π_d, π_d)

(1)

where the payoff entries satisfy the inequalities $\pi_a > \pi_c > \pi_d > \pi_b$.

The scheme of possible results of payoffs is as follows. If player I decides to cooperate, I can receive the second best possible outcome π_c if the opponent II also cooperates, but I's attempt to cooperate is exposed to being exploited by II if II decides to defect. In the latter scenario, II would collect the best outcome of value π_a while leaving I with the lowest payoff π_b. If player I decides to defect, then this player is guaranteed not to obtain the lowest payoff, but at least an amount π_d if player II defects as well. If player II decided to cooperate, then I is taking advantage of the situation and obtaining the maximum benefit π_a.

Technically, we say that mutual defection is the Nash equilibrium of this game because there is no unilateral deviation that could make the deviating player earn more, while mutual cooperation is the Pareto optimal situation. Therefore, this game represents a social dilemma for the players: the individual choice of defection dominates the attempt to cooperate for any given choice of the opponent, which is not socially optimal. Why is this a dilemma? Because this game formalizes a conflict between the individual (the Nash equilibrium) and the collective (Pareto optimal) level of reasoning: if both players actually choose to defect, both of them generate a total payoff of $2 \times \pi_d$, which is by definition lower than the aggregate payoff if both of them coordinated in full cooperation, $2 \times \pi_c$.

The standard version of the prisoner's dilemma game is a one-shot strategic interaction with simultaneous moves by the opponents. This implies that both players make their own individual decision (whether to cooperate or not) without knowing what the opponent is choosing. Once both players have chosen their strategy, both actions become public and the payoffs are generated.

Each player reacts to his own belief or expectation on the opponent's intention, and as a consequence, the preferred action in the dilemma crucially depends on the way players form their beliefs about the opponent moves. Therefore, it is important to understand how beliefs and preferences do (or do not) influence each other in this decision-making process.[1]

2.2. Sequential prisoner's dilemma

The experiment conducted by Blanco et al. (2014) focuses on a variation of the Prisoner's Dilemma game discussed above: a sequential one. In Fig. 1, we show the game tree of the game played in this sequential experiment (b), and compare it to its standard (simultaneous) counterpart with equivalent payoffs (a). In the sequential version, the solution concept required is the

[1] See Blanco et al. (2014, Section 1) about possible correlations between preferences and beliefs in dilemmas with models of social preferences such as inequality aversion and reciprocal preferences.