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Quantum probability updating from zero priors (by-passing Cromwell's rule)

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HIGHLIGHTS

- Experimental validation of non-Bayesian updating in decision making.
- Quantum probability model of concrete decision making problem.
- Investigation of zero-priors trap in decision making.

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ABSTRACT

Cromwell's rule (also known as the zero priors paradox) refers to the constraint of classical probability theory that if one assigns a prior probability of 0 or 1 to a hypothesis, then the posterior has to be 0 or 1 as well (this is a straightforward implication of how Bayes' rule works). Relatedly, hypotheses with a very low prior cannot be updated to have a very high posterior without a tremendous amount of new evidence to support them (or to make other possibilities highly improbable). Cromwell's rule appears at odds with our intuition of how humans update probabilities. In this work, we report two simple decision making experiments, which seem to be inconsistent with Cromwell's rule. Quantum probability theory, the rules for how to assign probabilities from the mathematical formalism of quantum mechanics, provides an alternative framework for probabilistic inference. An advantage of quantum probability theory is that it is not subject to Cromwell's rule and it can accommodate changes from zero or very small priors to significant posteriors. We outline a model of decision making, based on quantum theory, which can accommodate the changes from priors to posteriors, observed in our experiments.

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1. Introduction

Probability theory is at the heart of our effort to understand the working of the human mind: As cognitive agents, we have to survive and flourish in an inherently uncertain world. Therefore, it is a reasonable hypothesis that the principles that guide cognition are those of a formal probability theory. The most widely employed and successful probability theory in cognitive science is classical probability (CP) or Bayesian theory. CP cognitive models have been

the basis for successful cognitive explanation for many aspects of behavior, including decision making, learning, categorization, perception, and language. Moreover, CP has provided a foundation for issues key to human existence, such as the question of rationality (for overviews see Griffiths, Chater, Kemp, Perfors, & Tenenbaum, 2010; Oaksford & Chater, 2007; Tenenbaum, Kemp, Griffiths, & Goodman, 2011). Yet there are some behavioral situations that prove problematic from a CP perspective and challenge the universal applicability of CP theory in cognition. The focus of this paper is one such situation, related to probabilistic updating.

Probabilistic updating concerns the rules for how we should revise our estimate for the probability of a hypothesis, given some new information. According to CP, our belief in different

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hypotheses should be updated using Bayes' law, which states that

$$p(H_i|D) = \frac{p(D|H_i) p(H_i)}{\sum_j p(D|H_j) p(H_j)}. \quad (1)$$

Here H_i is a particular hypothesis and D is the observed data. In the set-theoretical paradigm, Eq. (1) is equivalent to

$$p(H_i|D) = \frac{p(H_i \cap D)}{\sum_j p(H_j \cap D)}. \quad (1')$$

The term $p(H_i \cap D)$ is the joint probability of a hypothesis and the data. Bayes' rule is highly intuitive, indeed to the point that it is hard to envisage alternatives. Our research program has largely been concerned with exploring a particular alternative probabilistic framework, which reveals alternative intuitions about probabilistic inference.

One important characteristic of Bayes' law is its stringent linearity; updated probabilities are linearly dependent on the priors. Thus, Bayes' rule must satisfy a very important property: when the prior is zero, then regardless of the data that we observe, the posterior probability, $p(H_i|D)$ as calculated by Bayes' law, has to be zero as well. Likewise, when the prior is one, the posterior has to be one as well. In fact, the earliest documented instance of this problem goes back to the 17th century. Oliver Cromwell allegedly said to the members of the synod of the Church of Scotland "to think it possible that you may be mistaken" (Carlyle, 1855). The argument goes that (if one cares to stay open-minded) one should assign some small possibility to even the most improbable state of affairs. Otherwise, however much evidence is subsequently accumulated in favor of the zero-prior possibility, if one employs Bayes' law for probability updating, one will be trapped with all-zero posteriors. This observation has been called Cromwell's rule, but we can also call it a zero priors trap. But the problem is more general than just concerning prior hypotheses, which are entirely possible or impossible, since the ratio of the likelihoods is constrained by the ratio of the priors. This means that for initially very unlikely hypotheses, whatever the evidence we observe, it will be impossible to attain high posteriors. Behaviorally this seems unlikely and it is the purpose of the present paper to explore this intuition formally and empirically. Note, the validity of Bayesian updating has been questioned before, for example, by Van Wallendael and Hastie (Robinson & Hastie, 1985; Van Wallendael, 1989; Van Wallendael & Hastie, 1990) who noted that, upon receiving information about one hypothesis, people tend to revise only the corresponding probability and leave their other estimates untouched (so that the total fails to equal one).

A natural domain for testing Cromwell's rule is detective stories, where the criminal turns out to be someone completely unexpected (as in Dostoevsky's *The Brothers Karamazov*, where, in full compliance with CP rules, prior beliefs about the innocence of the actual criminal were so strong, that his complete confession together with other hard evidence failed to convince the court) or not even a person (as in Poe's *The Murder in the Rue Morgue*). We present a simple experimental paradigm, based on a crime mystery, where all suspects are listed beforehand. The information about the crime mystery that participants initially see makes some suspects impossible/extremely unlikely. We then provide additional information, which makes some of these initially impossible options likely. Do participants update their beliefs in a way that is inconsistent with Bayes' law? That is, do participants produce evaluations of posterior probabilities that exceed the prior probabilities by amounts that go beyond what is allowed by Bayes' law? We will demonstrate that the probabilities given by participants in the course of this crime-solving paradigm strongly violate Bayesian updating. In fact, in 20% of cases, participants

updated a prior of close to zero to a very high posterior, in a single step.

Seeking to obtain evidence against Bayes' law in decision making raises the question of whether there may be an alternative formal framework for understanding probability updating, for these situations where Bayes' law may be problematic. Are there principles for probabilistic updating that allow the impossible to become possible? The standard way to deal with such situations in CP theory is additive smoothing, but this is typically applied a posteriori. For example, after we identify a new possibility, we can reshuffle the prior probabilities adding a "pseudo-count" to all the priors, so that no option on the (a posteriori!) list has a zero prior. There are other approaches (Hoppe, 1984), such as the Pólya urn model, directly enforcing an increase in the probability of a less probable event, each time the more probable event happens. For example, starting with an urn containing only (or mostly) black balls, we add a white ball each time a black ball is drawn. But what about options (e.g., balls of some new color) that are considered completely impossible at the outset? Yet a third possibility involves Shafer's representation of belief states (Shafer, 1976) where possible options are grouped (so that one has beliefs about the groups rather than individual hypotheses). In Section 4 we briefly consider whether this approach can address the zero priors paradox.

One probabilistic framework that is not constrained by Cromwell's rule is quantum probability theory (QPT). In QPT, the rule for updating probabilities, called the von Neumann–Lüders' rule, allows arbitrarily large increases in posterior probabilities, even if the priors are zero, when there are discontinuous changes in belief states (resulting from measurements).

We call QPT the rules for how to assign probabilities to events from quantum mechanics, without any of the physics. QPT is in principle applicable in any situation where there is a need to formalize uncertainty. The motivation to employ QPT in psychology exactly concerns situations, where the prescription from CP is at odds with human behavior or intuition. For example, in decision making, such situations correspond to findings that human behavior violates CP principles, such as the law of total probability or the commutativity of conjunctions (Conte et al., 2007; Croson, 1999; Hofstadter, 1983; Shafir & Tversky, 1992; Trueblood & Busemeyer, 2011; Tversky & Kahneman, 1983; Tversky & Shafir, 1992). Corresponding QPT cognitive models have achieved a measure of descriptive success, usually through assumptions that particular variables or questions are incompatible (overviews in Busemeyer & Bruza, 2012; Haven & Khrennikov, 2012; Khrennikov, 2003; Khrennikov, 2010; Pothos & Busemeyer, 2013). Note, a uniform assumption in such models is that they are all effectively hidden variables models, that it, that there is no 'true' quantum structure in cognition (Yearsley & Pothos, 2014; cf. Dubois & Lambert-Mogiliansky, 2015).

QPT provides an entirely different (compared to the standard CP one) framework for probabilistic inference, where events and probabilities are represented by vectors in a Hilbert space and Hermitian operators. We shall outline the instruments from QPT relevant to Cromwell's rule in the next section. In closing the introduction, it is worth noting that in exploring alternative probabilistic frameworks, QPT is not the only choice, and there are options that are even more non-classical, compared to QPT. For example, there are probability models that account for "negative probabilities" regularly encountered both in physics and cognitive psychology (Acacio de Barros, 2014; Acacio de Barros & Oas, 2014; see also Narens, 2015).

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