ARTICLE IN PRESS

Journal of Mathematical Psychology ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmp

Statistical and subjective interpretations of probability in quantum-like models of cognition and decision making

Emmanuel Haven [a], Andrei Khrennikov [b,*]

[a] School of Management and IQSCS, University of Leicester, UK

[b] International Center for Mathematical Modeling in Physics and Cognitive Sciences, Linnaeus University, Växjö, Sweden

HIGHLIGHTS

- Quantum and classical probability foundations are reviewed.
- Quantum-like approach to Decision making is presented.
- Subjective and statistical interpretations of quantum probability are analyzed.

ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Classical and quantum probability
Subjective and statistical probabilities
Interpretation
Decision making
Probability update
QBism
Växjö interpretation

ABSTRACT

The paper starts with an introduction to the basic mathematical model of classical probability (CP), i.e. the Kolmogorov (1933) measure-theoretic model. Its two basic interpretations are discussed: statistical and subjective. We then present the probabilistic structure of quantum mechanics (QM) and discuss the problem of interpretation of a quantum state and the corresponding probability given by Born's rule. Applications of quantum probability (QP) to modeling of cognition and decision making (DM) suffer from the same interpretational problems as QM. Here the situation is even more complicated than in physics. We analyze advantages and disadvantages of the use of subjective and statistical interpretations of QP. The subjective approach to QP was formalized in the framework of Quantum Bayesianism (QBism) as the result of efforts from C. Fuchs and his collaborators. The statistical approach to QP was presented in a variety of interpretations of QM, both in nonrealistic interpretations, e.g., the Copenhagen interpretation (with the latest version due to A. Plotnitsky), and in realistic interpretations (e.g., the recent Växjö interpretation). At present, we cannot make a definite choice in favor of any of the interpretations. Thus, quantum-like DM confronts the same interpretational problem as quantum physics does.

1. Introduction

Recently the mathematical formalism of quantum mechanics ('QM' henceforth), especially the apparatus of quantum probability ('QP' henceforth), started to be widely used outside of physics for the modeling of cognition and decision making ('DM' henceforth) in psychology, psychophysics, economics, finance, political science and the wider social sciences, see the basic monographs (Asano, Khrennikov, Ohya, Tanaka, & Yamato, 2015; Bagarello, 2012; Busemeyer & Bruza, 2012; Ezhov & Berman, 2003; Haven & Khrennikov, 2013; Khrennikov, 2010) and the recent review articles (Busemeyer, Wang, Khrennikov, & Basieva, 2014; Plotnitsky, 2014) and references therein; as well as a selection of some recent publications relevant to probabilistic foundations (Aerts, Sozzo, & Tapia, 2012; Aerts, Sozzo, & Veloz, 2015; Atmanspacher & Filk, 2014a,b; Atmanspacher, Haven, Kitto, & Raine, 2014; de Barros & Oas, 2014, 2015; de Barros & Suppes, 2009; Sozzo, 2015). Such models can be called *quantum-like* to distinguish them from genuine quantum physical models. In quantum-like models we explicitly do not refer to quantum physical processes which (may) take place in biological systems, in particular, in the brains of decision makers. Our modeling is based on the *quantum-like paradigm* (see Khrennikov, 2010): the process of DM within bio-systems with a complex information structure (e.g., by humans) is described by QP. This paradigm has an empirical origin: there is plenty of

* Corresponding author.
E-mail address: Andrei.Khrennikov@lnu.se (A. Khrennikov).

http://dx.doi.org/10.1016/j.jmp.2016.02.005

probabilistic data available, e.g., in cognitive psychology and psychophysics which exhibit the violation of the basic laws of classical probability ('CP' henceforth), e.g. the *formula of total probability* ('FTP' henceforth) (see, e.g., Busemeyer & Bruza, 2012; Khrennikov, 2010) or the *Bell inequality* (see, e.g., Conte et al., 2008; Khrennikov, 2010). Violations of the laws of classical probability theory by quantum physical systems were discussed by many authors (see, e.g., Feynman & Hibbs, 1965, or Khrennikov, 2009). This situation is well modeled by QP based on Born's rule connecting complex probability amplitudes (complex state vectors, wave functions) with real probabilities. One could make the argument that it could be useful to try to model similar violations of classicality outside of physics with the aid of the same calculus. However, this apparent similarity does surely not guarantee that the formalism which worked so well in one domain of science, in physics, will work as well in other domains. Its fruitfulness can be justified by successful applications. We remark that the situation does not differ so much from physics. QM is held in very high esteem because it works so well. On the other hand, the project on the justification of the impossibility of its reduction to classical statistical models (see, e.g., von Neumann, 1955, or Bell, 1987), still has not been completed (Khrennikov, 2008, 2010).[1]

Applications of the quantum formalism and, in particular, QP to model cognition and DM can be characterized as really successful (see Asano et al., 2015; Bagarello, 2012; Busemeyer & Bruza, 2012; Busemeyer et al., 2014; Ezhov & Berman, 2003; Haven & Khrennikov, 2013; Khrennikov, 2010; Plotnitsky, 2014) for various studies. At the same time one has to be cautious. One cannot expect that the whole body of QM would be useful for such applications. Moreover, it may happen that some cognitive or social phenomena would not be covered completely by the standard quantum formalism (cf. Khrennikov, Basieva, Dzhafarov & Busemeyer, 2014). It may well be that more general probabilistic models have to be developed (see Khrennikov, 2010).

We remark that although QM works very well, its theoretical and philosophic justification is far from complete. In particular, QM suffers from the problem of interpreting a quantum state (wave function) (see, for example, Khrennikov, 2009; Plotnitsky, 2006, 2009). The present situation is characterized by a huge diversity of interpretations and this cannot be considered as acceptable. Since QM is about probabilities (it does not predict the individual outputs of measurements), the problem of the interpretation of a quantum state is very closely related to the problem of the interpretation of a probability. In this paper we analyze the probability interpretation dimension of QM in connection to DM and to applications of QM's cognitive psychology. Of course, the state interpretation problem is not reduced to the interpretation of probability given by Born's rule. Thus, in this paper we shall treat the problem of an interpretation of QM only partially.

In any scientific theory one has to distinguish the formalism and its interpretation. The mathematical formalism of modern classical probability theory is based on measure theory (see Kolmogorov, 1933). However, it is interesting (and it maybe not so well known) that Kolmogorov not only developed the commonly used mathematical formalism of probability theory (including purely mathematical contributions such as Kolmogorov's theorem on the existence of the probability measure for a stochastic process and the strong law of large numbers), but he also endowed his theory with a special interpretation of probability, i.e. the Kolmogorov interpretation. Thus, just as in any theory, in Kolmogorov's theory one has to distinguish between the mathematical formalism and its interpretation. Besides the genuine Kolmogorov interpretation, his formalism can be interpreted in different ways. Among the huge variety of interpretations of probability, we point to two of the most known and applicable interpretations:

- **ST** statistical interpretation (Feller, 1968; Khrennikov, 2009; Kolmogorov, 1933; Plotnitsky, 2009; Rocchi, 2014; von Mises, 1957);
- **SUB** subjective (Bernardo & Smith, 1994; de Finetti, 1990; Ramsey, 1931; Rocchi, 2003, 2014; Savage, 1954).

ST: probability is a characteristic of a "mass phenomenon, or a repetitive event, or simply a long sequence of observations (see von Mises, 1957). Here probability cannot be assigned to an individual event. The condition of the event's repeatability (in theory infinite repeatability) is crucial. Numerically, probability is defined as the limit of frequencies (in von Mises' theory this is the definition of probability and in Kolmogorov's theory it is a consequence of the law of large numbers).

SUB: probability is assigned to an individual event A and it represents the degree of the personal belief in the non/occurrence of A. Thus, such probability is private and individual.

Now we want to couple the interpretations of a quantum state and the corresponding probability given by Born's rule. This coupling leads to two important interpretations of a quantum state:

- **STQ** statistical (ensemble) interpretation (Bohr, Pauli, Dirac, von Neumann, Einstein, Schrödinger, de Broglie, Bohm, Margenau, Ballentine)[2] (see, e.g., Khrennikov, 2009; Plotnitsky, 2006, 2009);
- **QBism** quantum Bayesian (subjective) interpretation (see, e.g., Fuchs, 2011; Fuchs & Schack, 2013, 2015).

STQ can be characterized by a diversity of 'sub-interpretations' depending on whether the results of observations can be treated independently of the measurement procedures or not (the problem of realism in QM). **QBism** was created recently and it has yet just one version. As we can see from the **STQ**-list, this interpretation dominates in the quantum community. In terms of recent contributions to its development we can mention the *Växjö interpretation* (see Khrennikov, 2002); the realist contextual statistical interpretation; the *statistical Copenhagen interpretation* invented by A. Plotnitsky[3] and the non-realist statistical interpretation. At the same time, the recent quantum information revolution stimulated the dissemination of **QBism**. However, it is still considered as an exotic 'non-physical' interpretation of QM.[4]

Now, suppose one applies QP to model the DM-process, e.g., in psychology, psychophysics or economics. She/he is immediately confronted with the cognitive/mental version of the problem of the interpretation of quantum states and probabilities: the problem which was not solved in quantum physics and was 'imported' from it to cognitive science, DM, psychology or psychophysics. Moreover, novel applications induce novel interpretational issues. Our aim is to analyze the specifics of the use of **STQ** and **QBism** to model cognition and DM. The problem is very complex and at the moment we are only able to present some reasons in favor of and against each of these interpretations. We hope that our analysis will stimulate the further emergence of foundational studies on the problem of the interpretations of mental states (belief states) and the corresponding probabilities in QP-modeling of DM and problem solving.

[1] The von Neumann theorem was strongly criticized for its un-physical assumptions, by Margenau, Bell and Ballentine. Experimental verification of a violation of Bell's inequality is a very challenging project, since it is very difficult to perform the loophole free experiment producing statistically acceptable data (see, e.g., Khrennikov, Ramelow et al., 2014) for analysis and Hensen et al. (2015) for the most recent success in this area.

[2] It is interesting that very different interpretations of QM can keep the same interpretation of probability. For example, both the Copenhagen interpretation and the de Broglie–Bohm interpretation treat probability statistically.

[3] It was presented in his talk at the conference "Quantum Theory: from Foundations to Technologies", Växjö -2015.

[4] QBism is often labeled as one of the neo-Copenhagen interpretations of QM. This is a totally wrong viewpoint on QBism (see, for example, Mermin, 2014).

Please cite this article in press as: Haven, E., & Khrennikov, A., Statistical and subjective interpretations of probability in quantum-like models of cognition and decision making. Journal of Mathematical Psychology (2016), http://dx.doi.org/10.1016/j.jmp.2016.02.005