ARTICLE IN PRESS

Journal of Mathematical Psychology ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmp

From ambiguity aversion to a generalized expected utility. Modeling preferences in a quantum probabilistic framework

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HIGHLIGHTS

- We develop a general probabilistic framework to model human decisions under uncertainty.
- Our quantum theoretical model faithfully represents different sets of data on Ellsberg and Machina paradoxes.
- Our approach captures fundamental aspects of ambiguity, where traditional approaches are problematical.
- Our approach opens the way toward a quantum-based state-dependent generalization of expected utility theory.

ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Expected utility theory
Ellsberg and Machina paradoxes
Quantum probability
Quantum modeling

ABSTRACT

Ambiguity and ambiguity aversion have been widely studied in decision theory and economics both at a theoretical and an experimental level. After Ellsberg's seminal studies challenging subjective expected utility theory (SEUT), several (mainly normative) approaches have been put forward to reproduce ambiguity aversion and Ellsberg-type preferences. However, Machina and other authors have pointed out some fundamental difficulties of these generalizations of SEUT to cope with some variants of Ellsberg's thought experiments, which has recently been experimentally confirmed. Starting from our quantum modeling approach to human cognition, we develop here a general probabilistic framework to model human decisions under uncertainty. We show that our quantum theoretical model faithfully represents different sets of data collected on both the Ellsberg and the Machina paradox situations, and is flexible enough to describe different subjective attitudes with respect to ambiguity. Our approach opens the way toward a quantum-based generalization of expected utility theory (QEUT), where subjective probabilities depend on the state of the conceptual entity at play and its interaction with the decision-maker, while preferences between acts are determined by the maximization of this 'state-dependent expected utility'.

1. Introduction

Increasing experimental evidence in cognitive psychology seems to confirm that human decisions do not generally obey the constraints of classical logic and probability theory. The deviations that have been experimentally detected up to now can be roughly divided into two groups, 'probability judgment errors' (e.g., 'conjunction and disjunction fallacy' Morier & Borgida, 1984; Tversky & Kahneman, 1983, 'concept categorization' Hampton, 1988a,b), 'unpacking effects' (Fox & Tversky, 1998) and 'decision-making errors' (e.g., 'disjunction effect' Tversky & Shafir, 1992, 'Allais, Ellsberg and Machina paradoxes' Allais, 1953; Ellsberg, 1961; Machina, 2009). In particular, these 'fallacies', or 'errors', challenge the classical vision of probability theory that can be traced back to the axiomatization proposed by Kolmogorov in the thirties (Kolmogorov, 1933). When applied to decision theory and economics, this classical approach to probability has produced a unified normative axiomatic theory to model human decisions under uncertainty, which has been predictively successful for years since the forties, 'expected utility theory' (EUT) (Savage, 1954; von Neumann & Morgenstern, 1944). By following Knight's original distinction of the different forms of uncertainty (Knight, 1921), one nowadays

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http://dx.doi.org/10.1016/j.jmp.2016.02.007

distinguishes between 'risk', that is, 'uncertainty about known probabilities', and 'ambiguity', that is, 'uncertainty about unknown probabilities'. And also EUT takes a different mathematical formulation, depending on whether one considers situations in which risk is present, 'objective expected utility theory' (OEUT) (von Neumann & Morgenstern, 1944), or situations in which ambiguity is present, 'subjective expected utility theory' (SEUT) (Savage, 1954). It should however be noticed that the latter situations have manifold implications in real-life decisions, from financial asset pricing to marketing choices, portfolio management, medical treatment decisions, as subjective probabilities frequently appear in everyday decisions, in the form of 'beliefs about likelihood of events'.

The first experimental challenge to OEUT was identified by Alain Allais in the fifties (Allais, 1953). Coming to SEUT, its drawbacks arose instead with the so-called 'Ellsberg urns' (Ellsberg, 1961). Daniel Ellsberg presented in 1961 a series of thought experiments with urns and balls of different colors, e.g., 'two-color example' and 'three-color example', where he predicted that real human decisions would have refuted the predictions of SEUT. Ellsberg suggested that people prefer to take actions associated with events over known rather unknown probabilities or, in other words, 'people prefer known versus unknown probabilities'. This conservative attitude was called 'ambiguity aversion' by Ellsberg, and it violates a specific axiom of SEUT, the 'Sure-Thing principle'—a violation of this principle is also observed in other decision-making errors, such as the disjunction effect (Tversky & Shafir, 1992). Ambiguity aversion has been systematically confirmed by several cognitive experiments collected in the last forty years (see, e.g., Camerer & Weber, 1992; Einhorn & Hogarth, 1986; Fox & Tversky, 1995; McCrimmon & Larsson, 1979; see also the exhaustive review by Machina & Siniscalchi, 2014), but the one by Slovic and Tversky (1974), who found an 'ambiguity seeking' attitude. Both ambiguity aversion and ambiguity attraction are however incompatible with the predictions of SEUT, which led many scholars to look for theoretical alternatives that could accommodate the behavior observed in Ellsberg experiments. These alternatives have generally a normative status, that is, they describe what people should not, not what people actually do, and have different names and scopes, for example, 'expected utility with multiple priors' (Gilboa & Schmeidler, 1989), 'Choquet expected utility theory' (CEUT) (Schmeidler, 1989), 'smooth ambiguity preferences model' (Klibanoff, Marinacci, & Mukerji, 2005), 'variational preference model' (Maccheroni, Marinacci, & Rustichini, 2006), and 'cumulative prospect theory' (Tversky & Kahneman, 1992), within Tversky–Kahneman theory of human heuristics and bias (Tversky & Kahneman, 1974).

These generalizations of SEUT were seriously challenged by two thought experiments recently presented by Mark Machina (2009), the '50:51 example' and the 'reflection example' (Baillon, L'Haridon, & Placido, 2011; Machina, 2009). In particular, Machina introduced a new element, 'informational symmetry', which violates an axiom of CEUT, called 'tail separability', exactly as ambiguity aversion violates the Sure-Thing principle. Without entering into the technical details of the violation (some aspects of it will be presented in Section 2), we want to emphasize an implicit assumption of SEUT that is weakened in the above generalizations. SEUT assumes that a 'single Kolmogorovian probability distribution over a single σ-algebra of events is defined which models subjective probabilities' in human decisions. Departure from this assumption will become crucial in our approach, as we will see in Section 3.

We have worked in the last years on the identification of quantum structures in cognition and the mathematical modeling of decision-making under uncertainty, obtaining significant results in the explanation of the so-called human probability judgment errors in terms of genuine quantum aspects (emergence, entanglement, indistinguishability, interference, superposition) (Aerts, 2009; Aerts, Broekaert, Gabora, & Sozzo, 2013; Aerts & Gabora, 2005a,b; Aerts, Gabora, & Sozzo, 2013; Aerts, Sozzo, & Veloz, 2015a,b; see also Busemeyer & Bruza, 2012; Haven & Khrennikov, 2013; Pothos & Busemeyer, 2009; Pothos & Busemeyer, 2013; Wang, Solloway, Shiffrin, & Busemeyer, 2014). For what explicitly concerns ambiguity and ambiguity aversion, we have worked out a quantum theoretical framework to model both the Ellsberg and the Machina paradox situations (Aerts, Sozzo, & Tapia, 2012, 2014). Our approach rests on two fundamental observations, as follows.

(i) In an Ellsberg/Machina-type decision-making process, the agent's choice is actualized as a consequence of an interaction with the cognitive context, exactly as in a quantum measurement process the measurement outcome is actualized as a consequence of the interaction of the measured particle with the measuring apparatus. Therefore, in cognitive entities, as well as in microscopic quantum entities, measurements do not reveal preexisting values of the observed properties but, rather, they actualize genuine potentialities. Kolmogorovian probability can only formalize lack of knowledge about actualities, hence it is generally not able to cope with such a decision-making process. We have proven that this is possible in Ellsberg/Machina-type decisions by using a complex Hilbert space and representing probability measures by means of 'projection valued measures' on this complex Hilbert space. A projection valued measure is essentially different from a single Kolmogorovian probability measure, since the latter is a σ-algebra valued measure, whilst the former is not, due to lack of distributivity (Aerts et al., 2012).

(ii) The above notion of ambiguity is completely compatible, both at a mathematical and an intuitive level, with the representation of states of cognitive entities as vectors of a Hilbert space. Indeed, just like in quantum theory the state vector incorporates the 'quantum uncertainty' of a microscopic particle, also in an Ellsberg/Machina-type situation, the agent's subjective preference toward ambiguity is naturally formalized by representing the conceptual situation the agent is confronted with by means of such a Hilbert space vector. In our approach, ambiguity aversion is only one of the conceptual landscapes surrounding the decision-maker's choice in a situation where ambiguity is present (Aerts et al., 2014). This representation is compatible with the experimental findings confirming Ellsberg's prediction about the human attitude toward ambiguity, but also with some recent experiments where such attitude is more controversial (see, e.g., Binmore, Stewart, & Voorhoeve, 2012; Slovic & Tversky, 1974); see also the review (Machina & Siniscalchi, 2014).

We develop in Section 3.1 an amended and updated theoretical framework, where subjective probabilities and preferences in the Ellsberg paradox situation are modeled by using the mathematical formalism of quantum theory, which is briefly summarized in the Appendix. Further, we show in the same section that our quantum theoretical model successfully represents an experiment on the Ellsberg three-color example by ourselves, thus confirming Ellsberg-type preferences (Aerts et al., 2012). This theoretical framework can be adapted to model probabilities and preferences in the Machina paradox situation. We show this result in Section 3.2, where we also represent an experiment on the Machina reflection example by L'Haridon and Placido (2010), thus confirming Machina-type preferences.

Our quantum theoretical framework opens the way to a 'contextual quantum-based generalization of EUT' (QEUT), where human preferences also depend on the conceptual, not only the physical, state encoding the potential effects of the cognitive context. These effects include, in particular, subjective preferences toward ambiguity and ambiguity aversion.

Please cite this article in press as: Aerts, D., & Sozzo, S., From ambiguity aversion to a generalized expected utility. Modeling preferences in a quantum probabilistic framework. Journal of Mathematical Psychology (2016), http://dx.doi.org/10.1016/j.jmp.2016.02.007