Swarm and Evolutionary Computation 15 (2014) 27–37

Contents lists available at ScienceDirect

Swarm and Evolutionary Computation

journal homepage: www.elsevier.com/locate/swevo

Regular Paper

An efficient genetic algorithm for multi-objective solid travelling salesman problem under fuzziness

Chiranjit Changdar[a,*], G.S. Mahapatra[b], Rajat Kumar Pal[c]

[a] Department of Computer Science, Raja N.L. Khan Women's College, Midnapore, 721102, West Bengal, India
[b] Department of Mathematics, National Institute of Technology, Puducherry, Karaikal 609605, India
[c] Department of Computer Science and Engineering, University of Calcutta, Calcutta 700 009, West Bengal, India

ARTICLE INFO

Article history:
Received 2 April 2013
Received in revised form
3 October 2013
Accepted 8 November 2013
Available online 26 November 2013

Keywords:
Travelling salesman problem
Possibility
Necessity
Refining operation
Multi-objective
Genetic algorithm

ABSTRACT

In this paper, we have presented a multi-objective solid travelling salesman problem (TSP) in a fuzzy environment. The attraction of the solid TSP is that a traveller visits all the cities in his tour using multiple conveyance facilities. Here we consider cost and time as two objectives of the solid TSP. The objective of the study is to find a complete tour such that both the total cost and the time are minimized. We consider travelling costs and times for one city to another using different conveyances are different and fuzzy in nature. Since cost and time are considered as fuzzy in nature, so the total cost and the time for a particular tour are also fuzzy in nature. To find out Pareto-optimal solution of fuzzy objectives we use fuzzy possibility and necessity measure approach. A multi-objective genetic algorithm with cyclic crossover, two-point mutation, and refining operation is used to solve the TSP problem. In this paper a multi-objective genetic algorithm has been modified by introducing the refining operator. Finally, experimental results are given to illustrate the proposed approach; experimental results obtained are also highly encouraging.

1. Introduction

The TSP is a classical combinatorial optimization problem, where the problem is to find a shortest possible tour through a set of n vertices such that each vertex is visited exactly once except for the starting vertex. Since the tour ends at the starting vertex, this problem is known to be NP-hard, and cannot be solved exactly in polynomial time [20,19]. In the existing literature on TSP, it is implicitly assumed that the travelling cost from one node to another is fixed, i.e., crisp in nature [4,3]. Different types of TSPs have been solved by the researchers during the last two decades. In TSP with precedence constraint [24], there exists an order in which the vertices are to be visited. In asymmetric TSP [23], the cost of travelling from vertex (node/city) v_i to v_j is not equal to the cost of travelling from vertex v_j to v_i. In stochastic TSP [22,2], each vertex is visited with a given probability and the goal is to minimize the expected distance/cost of a priori tour. In the TSP with time windows [10], each vertex is visited within a specified time window. In double TSP [30], the targets can be reached by two sales persons operating in parallel.

The travelling cost and speed change in different time and weather or traffic circumstance. Therefore, the travelling time and cost between two cities cannot be estimated in such a circumstance. The decision maker introduces tolerance to handle the vagueness in cost and time estimation of TSP. This leads to use of fuzzy set which enables us to make decisions based on vague or imprecise data. Also travelling cost from one node to another depends on the conveyance used for travelling. It also varies on a daily basis and hence it is better to consider the costs of a TSP as fuzzy in nature. Here the possibility of error is less as these estimations are based upon experts' opinion. Also the problem is desirable to be formulated in such a way that the salesman can visit one city to another using different conveyance. Though it is normally practiced by salesmen, few researchers consider the TSP with different conveyance facilities. Again time required for travelling using different conveyance in different paths is different. So the goal of a TSP is multiple at the present day rather than only single objective problem.

Algorithms for TSP: In existing literature besides exact methods, meta-heuristics, local search, and hybrid algorithms of optimization and searching approaches are applied to solve TSPs. The exact methods including cutting plane [9], LP relaxation [4], branch and bound [28], branch and cut [29], etc. have been used. However, very few TSPs can be solved by exact methods. On the other hand, several problems have been solved using heuristics or soft computing based techniques such as simulated annealing [1], local search [15], hybrid algorithm [10], tabu search [18], and genetic algorithm [26]. Kesen and Güngör [17], and Kesen et al. [16] presented scheduling of virtual manufacturing cells using the

* Corresponding author.
E-mail address: chiranjit_changdar@yahoo.co.in (C. Changdar).

2210-6502/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.swevo.2013.11.001

genetic algorithm based heuristic method; there are some similarity with the solid TSP. Majumder and Bhunia [23] solved asymmetric travelling salesman problem with imprecise travel times using a genetic algorithm.

Significance of the problem: In solid TSP there are different conveyance facilities to travel from one city to another city, and that is the main attraction of this model. If we consider a real life situation, such that a medical representative (MR) travels/meets some doctors of different cities within a time limit, then he must choose a Pareto-optimal solution for which the MR must cover/meet all the doctors within the time limit and minimum travel cost. But in some of the cases time limit is not a factor but travelling allowances is limited, then the MR selects such a solution (among all the Pareto-optimal solutions) which is matched to the MR's travelling allowance with a minimum time, i.e., a compromised solution for the trade-off has to be chosen by him. In solving the above mentioned situation this model is effective. In an existing literature many researchers implicitly assumed that travelling cost from one node to another is fixed, i.e., crisp in nature. But travelling cost and time from one node to another depends on the conveyance used for travelling.

TSP attracted and still attracts a large variety of research efforts due to its adaptability to real case conditions. In this paper, the researchers associate the solid TSP in an uncertain situation which has been modeled using fuzzy numbers. Fuzzy number is also effective as the travel time, normally is not fixed, or follows probabilistic nature. Due to this reason it is better to model the costs of a TSP as fuzzy numbers. It is less error prone as these estimations are based on experts' opinion.

In this paper, we consider a solid TSP, where a salesman can utilize different conveyances to travel from one city to another. Costs and times for travelling using different conveyances from one city to another are different. The salesman should optimize total travel time along with the total travel cost. Fuzzy cost and time may use to make the problem more realistic. Our problem is more complicated for impreciseness of parameters and multi-objective formation. Fuzziness of the cost and time leads to fuzzy total cost and fuzzy total time of a tour. As optimization of fuzzy objective is not well defined it is very difficult to find Pareto-optimal paths of the stated problem. Due to this complexity, we use fuzzy possibility/necessity based approach for fuzzy objective functions. A multi-objective genetic algorithm with cyclic crossover, two-point mutation, and refinement (refining) operation finds the Pareto-optimal paths for the proposed problem. Finally, numerical examples are illustrated to support of our proposed approach for fuzzy TSP.

2. Prerequisite mathematics

Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers with membership functions $\mu_{\tilde{a}}(x)$ and $\mu_{\tilde{b}}(x)$, respectively. Then according to Zadeh [36] and Dubois et al. [7], the fuzzy possibility and necessity is defined as follows:

$$Pos(\tilde{a} * \tilde{b}) = sup\{min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)); x, y \in \Re, x * y\}, \quad (1)$$

$$Nes(\tilde{a} * \tilde{b}) = 1 - pos\overline{(\tilde{a} * \tilde{b})}, \quad (2)$$

where $*$ is any one of the relations $>, <, =, \le, \ge$, and $\Re$ represents a set of real numbers. Here the abbreviation *Pos* and *Nes* represent possibility and necessity, respectively.

Triangular fuzzy number (*TFN*): A TFN $\tilde{a} = (a_1, a_2, a_3)$ has three parameters a_1, a_2, and a_3, where $a_1 < a_2 < a_3$ as shown in Fig. 1, and it is characterized by the membership function $\mu_{\tilde{a}}(x)$ as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

According to the above definition of TFN, following lemmas can easily be derived [6,21,36]. If $\tilde{a}$, $\tilde{b} \subseteq \Re$ and $\tilde{c} = f(\tilde{a}, \tilde{b})$, where $f : \Re \times \Re \to \Re$ is a binary operation, then the membership function $\mu_{\tilde{c}}$ of $\tilde{c}$ is defined as follows:

$$\mu_{\tilde{c}}(z) = \sup\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)), x, y \in \Re \text{ and } z = f(x, y)\} \ \forall \ z \in \Re \quad (4)$$

Lemma 1. *If $\tilde{a} = (a_1, a_2, a_3)$ be a TFN with $0 < a_1$ and b is a crisp number, then $pos(\tilde{a} < b) \ge \alpha$ iff $b - a_1 / a_2 - a_1 \ge \alpha$, and $nes(\tilde{a} < b) \ge \alpha$ iff $a_3 - b / a_3 - a_2 \le 1 - \alpha$.*

Lemma 2. *If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be TFNs with $0 < a_1$ and $0 < b_1$, then $pos(\tilde{a} > \tilde{b}) \ge \alpha$ iff $(a_3 - b_1)/(a_3 - a_2 + b_2 - b_1) \ge \alpha$, and $nes(\tilde{a} > \tilde{b}) \ge \alpha$ iff $(b_3 - a_1)/(a_2 - a_1 + b_3 - b_2) \le 1 - \alpha$.*

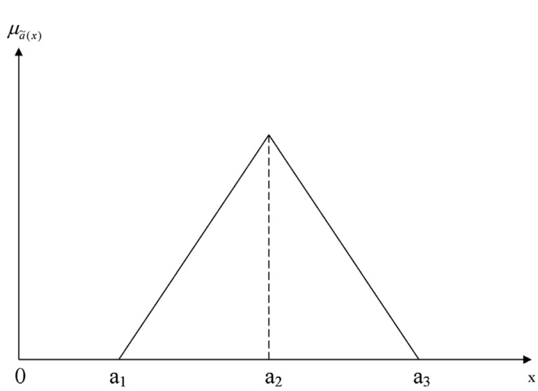

Fig. 1. Triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$.

3. Formulation of solid TSP in crisp and fuzzy environment

In this section, we present TSP in both crisp and fuzzy environments. First we present TSP in a crisp environment that is with crisp cost and time. Next we present mathematical formulation of the TSP with fuzzy cost and time.

3.1. Solid TSP with crisp cost and time

In a solid TSP, the salesman determines a tour program to travel n cities exactly once. To travel from one city to another, the salesman can use any one of the m types of available conveyance. The time required for travelling from one city to another using different conveyances is different. In this tour, the salesmen starts from a city and visits all the cities exactly once using suitable conveyances available at the cities and comes back to the starting city. In the tour, the goal of the salesman is twin as follows:

- Minimize the total cost.
- Minimize the total time.

Let $c(i, j; k)$ and $t(i, j; k)$ be the cost and time, respectively, for travelling from ith city to jth city using kth conveyance. The salesman has to determine a complete tour $(x_1, x_2, \ldots, x_n, x_1)$ and the corresponding conveyance types $(v_1, v_2, \ldots, v_n)$ to be used for the tour, where $x_i \in \{1, 2, \ldots, n\}$ and $v_i \in \{1, 2, \ldots, m\}$ for $i = 1, 2, \ldots, n$ and all x_i are distinct. Then the problem can be mathematically formulated as follows.