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Identifying learning difficulties with fractions: A longitudinal study of student growth from third through sixth grade

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ABSTRACT

The present longitudinal study examined growth in fraction knowledge between third and sixth grades ($N = 536$). Students were administered fraction concepts and procedures measures twice yearly through sixth grade. Analyses revealed empirically distinct growth classes on both measures. Of particular interest were students who started low and made little progress after three years of instruction in fractions, compared to those who started low but made good progress. Poorer language, attention, whole number line estimation, and calculation fluency in third grade significantly increased the odds of membership in a low-growth trajectory class for fraction concepts, while poorer attention and calculation fluency predicted membership in a low-growth trajectory class for fraction procedures. Students classified as receiving special education services in school, many of whom had diagnosed learning disabilities, were 2.5 times more likely to experience low growth in fraction concepts than their peers who were not receiving special education and 11.5 times more likely to experience low growth in fraction procedures. Students with persistent difficulties in fraction knowledge also were much less likely to meet state standards on a mathematics achievement test, portending problems in more advanced mathematics.

1. Introduction

Mathematics proficiency is important to educational, economic, and personal success in today's world. Mathematics achievement in high school predicts college matriculation and graduation (Murnane, Willett, & Levy, 1995), which opens myriad opportunities, especially in highly sought after STEM professions (National Mathematics Advisory Panel (NMAP), 2008; Sadler & Tai, 2007). Unfortunately, only a small percentage of U.S. students possess the mathematics prerequisites to succeed in STEM careers (National Academy of Sciences, 2007).

Proficiency with fractions, in particular, helps students succeed in algebra, a gateway to STEM professions (National Mathematics Advisory Panel (NMAP), 2008; Siegler et al., 2012). Fraction knowledge also promotes everyday skills, such as keeping budgets, calculating mortgage rates, and doing home repairs. As such, students' difficulties with fractions must be a priority for education research.

Fractions are a crucial component of the U.S. mathematics curriculum between third and sixth grades (Council of Chief State School Officers & National Governors Association Center for Best Practices, 2010). Unfortunately, many students struggle with fractions during this foundational period (Fuchs et al., 2013; Hecht, Vagi, & Torgeson, 2007). Relative to whole number skills, little attention has been devoted to understanding students' growth in fraction knowledge at the critical juncture between later elementary and early middle school (Siegler, Thompson, & Schneider, 2011), although it has been suggested that many students with mathematics learning difficulties and disabilities in eighth grade are characterized by deep-seated weaknesses with rational numbers (Mazzocco & Devlin, 2008). To address this gap in the literature, the goal of the present study is to examine the typical trajectory of fraction learning from third through sixth grade and then identify subgroups of children who differed from that trajectory. Within these subgroups, we identified the prevalence of students with and without diagnosed learning disabilities. Importantly, we examined the characteristics and mathematics achievement outcomes of students who showed low growth in fraction knowledge, students who are most at risk for persistent mathematics learning difficulties.

2. Conceptual framework

Geary's (2004) conceptual framework for studying and identifying potential mathematics learning difficulties proposes that competency in any area of mathematics depends on conceptual understanding as well as knowledge of procedures. Broadly speaking, conceptual knowledge involves understanding the particular principles that govern a mathematical domain as well as understanding

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the relations between different pieces of knowledge in that domain; procedural knowledge relates to the ability to execute steps to solve a computational problem (Rittle-Johnson, Siegler, & Alibali, 2001). In the area of fractions, students must understand how the numerator and denominator work together to determine magnitudes and that two or more fractions can be ordered on a number line (Siegler et al., 2011). Students also must apply procedures for solving fraction arithmetic problems, such as finding the common denominator for adding and subtracting.

Facility with fraction concepts supports learning of fraction procedures and vice versa (Hallett, Nunes, & Bryant, 2010; Hecht & Vagi, 2010, 2012; Hecht, Close, & Santisi, 2003; Rittle-Johnson & Siegler, 1998; Siegler et al., 2012). Moreover, there are individual differences in the way students combine their knowledge of fraction concepts and procedures to solve problems. For example, Hecht and Vagi (2012) show that although knowledge of fraction concepts is most important to general fraction achievement, some students can use their relatively strong knowledge of fraction procedures to compensate somewhat for their weaker knowledge of fraction concepts. Thus, fraction concepts and procedures develop in relation to each other.

According to Geary's (2004) framework, multiple cognitive processes, in turn, support the acquisition of mathematics concepts and procedures and these processes are important for understanding the difficulties students encounter. Domain general cognitive processes include the central executive system, the language system, and the visual spatial system. The central executive controls the attentional processes needed to solve complex mathematics problems. Attentive behavior, or the ability to stay on task and attend to instruction, helps students gain skills in the mathematics classroom (Finn, Pannozzo, & Voelkl, 1995) and predicts mathematics outcomes (Fuchs et al., 2005). Working memory helps students store and manipulate numerical information in short-term memory (Hecht et al., 2003) and facilitates problem solving accuracy (Swanson, 2011). The visual spatial system involves nonverbal or spatial reasoning, which can help student mentally represent numerical magnitudes (Gunderson, Ramirez, Beilock, & Levine, 2012). Finally, the language system is important for understanding and communicating with relevant mathematical terms (e.g., "numerator", "denominator", "equivalence") and for solving word problems (Seethaler, Fuchs, Star, & Bryant, 2011).

Augmenting Geary's framework, more recent work suggests that numerical magnitude understanding appears to be a critical underlying structure for learning mathematics more generally and fractions in particular (Siegler & Lortie-Forgues, 2014). A key property that unites all real numbers is that they can be assigned specific locations on number lines (Siegler & Lortie-Forgues, 2014; Siegler et al., 2011). Accurate representation of whole number magnitudes is even associated with recalling answers to arithmetic problems, which at one time was viewed as primarily a verbal rote memorization process. For example, incorrect answers that are close in magnitude to the correct answer take longer to reject than errors that are further in magnitude from the correct answer (Ashcraft, 1982). Students who develop an understanding that all real numbers have magnitudes that are assigned to their own location on a number line are likely to have an advantage in learning fractions (Siegler et al., 2011).

Domain general processes and domain specific magnitude understanding both appear to make unique contributions to fraction knowledge (Hansen et al., 2015; Hecht et al., 2003; Jordan et al., 2013). Jordan et al. (2013) found that whole number line estimation makes a large independent contribution to fraction knowledge in fourth grade when controlling for domain general competencies. Using mediation analyses, Vukovic et al. (2014) showed that domain general abilities support the acquisition of fraction concepts in fourth grade via skill in whole number line estimation in second grade (although the researchers did not look at the role of these competencies in supporting fraction procedures). These findings suggest that domain general competencies influence the development of fraction knowledge through promoting development of intermediate whole number skills, which, in turn, influences children's fraction learning. However, studies have not examined predictors of growth in fraction knowledge (i.e., fraction concepts and procedures) throughout the course of fraction instruction between third and sixth grades nor have they identified characteristics of students with persistent fraction weaknesses, ones who have or are at high risk for broader mathematics difficulties.

3. Fraction difficulties

Students who have yet to learn fractions generally operate under the incorrect assumption that properties of whole numbers hold true for all numbers (sometimes called the "whole number bias"; Ni & Zhou, 2005; Siegler et al., 2011). However, when students encounter fractions, they must see that some of the properties of whole numbers do not apply. For example, multiple fractions can refer to the same location on the number line (2/4 is the same location as 1/2 and 4/8) and the same fraction can refer to very different sets of objects (1/2 may refer to parts of a whole or parts of a set). Moreover, the magnitudes of fractions do not change in consistent ways with the absolute values of the numerators and denominators (Schneider & Siegler, 2010). For example, five is greater than three, and 15 is greater than six, but 5/15 is less than 3/6. Additionally, procedures with fractions are not always consistent with whole number rules (e.g., multiplication of two fractions can yield an answer smaller than either multiplicand, while multiplication of whole numbers always produces a larger product). Errors on fraction computation problems may reflect misapplication of whole number principles to fractions, or, alternatively, older students who have been taught fraction operations with multiplication might confuse fraction addition and subtraction procedures with those for fraction multiplication, where it is correct to operate across the numerators and denominators (Newton, Willard, & Teufel, 2014). On the other hand, whole number and fraction operations share the same underlying conceptual structure (Alibali & Sidney, 2015). Understanding how many times the divisor goes into the dividend can help students see why eight divided by two is four as well as why eight divided by 1/2 is 16.

Students also need to develop an understanding of the density of rational numbers, that there is an infinite amount of fractional numbers between any two consecutive integers, such as zero and one. Smith, Solomon, and Carey (2005) assessed third to fifth graders' understanding of infinite divisibility and discovered that when children recognize that there are numbers between zero and one, they only name unit fractions, such as 1/2 and 1/4. This observation demonstrates that although some students understand that there are some numbers between zero and one, they do not recognize that there is a limitless amount of numbers between the two integers.

The development of numerical knowledge is often viewed as a segmented process, in which knowledge of whole numbers is acquired somewhat naturally and then later, fraction knowledge is acquired with much difficulty (Geary, 2006; Gelman & Williams, 1998). Siegler et al. (2011) propose that numerical development is a continuous process that involves a gradual refining of the definition of number; the whole number bias can be overcome by learning that the one property that unites all real numbers is that they can be assigned specific locations on number lines. Conceptual change, however, is challenging when new information about fractions seems incompatible with the student's existing framework (McMullen, Laakkonen, Hannula-Sormunen, & Lehtinen, 2014; Van Hoof, Janssen, Verschaffel, & Van Dooren, 2015; Vosniadou, 2014). Unfortunately, many students even in high school and