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Impacts of inquiry pedagogy on undergraduate students conceptions of the function of proof

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ABSTRACT

Mathematicians and mathematics educators agree that proof is an important tool in mathematics, yet too often undergraduate students see proof as a superficial part of the discipline. While proof is often used by mathematicians to justify that a theorem is true, many times proof is used for another purpose entirely such as to explain why a particular statement is true or to show mathematics students a particular proof technique. This paper reports on a study that used a form of inquiry-based learning (IBL) in an introduction to proof course and measured the beliefs of students in this course about the different functions of proof in mathematics as compared to students in a non-IBL course. It was found that undergraduate students in an introduction to proof course had a more robust understanding of the functions of proof than previous studies would suggest. Additionally, students in the course taught using inquiry pedagogy were more likely to appreciate the communication, intellectual challenge, and providing autonomy functions of proof. It is hypothesized that these results are a response to the pedagogy of the course and the types of student activity that were emphasized.

1. Introduction

Proof is the main component of pure mathematical practice (Rav, 1999) and is the heart of what most mathematicians do; however, the role of proof can vary depending on the context of its presentation or creation. For instance, one can use proof to validate a conjecture that was made during mathematical research, but when that proof is published or presented it is being used more to communicate an idea. Mathematics students may use proof not as much to validate a conjecture, but to explain why a particular statement is true or to shed light on the structure of mathematics, while mathematics instructors can use proof to teach techniques to students and to explain why certain statements are true. While we may agree as mathematicians and educators that proof fills the roles outlined above, our students may not appreciate how crucial proof is to the discipline or its many different facets. Even more concerning, students may perceive proof in mathematics as needed merely for justification (Weber, 2002) or as something required only by a teacher and unimportant to the nature of the discipline (Alibert and Thomas, 1991).

It is hypothesized that informing students about these additional functions of proof when teaching can be advantageous (Hanna, 1990; Knuth, 2002a) and that “the more fundamental function of explanation should be exploited to present proof as a meaningful activity to pupils” (de Villiers, 1990, p. 23). However, in most transition to proof courses, which would be the natural class in which to discuss such things, explicit instruction on the functions of proof is not a normal part of the curriculum and so may not be taught. Thus, it is crucial to determine which functions of proof students in a transition to proof course, where they will likely be exposed to proof-writing for the first time in their studies, may already understand and appreciate, as well as to explore pedagogical ways to increase appreciation of the many roles of proof without spending class time to teach such things. Therefore, the present study aimed

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to identify students' perceptions of the function of proof before and after a transition to proof course as well as to determine if an inquiry pedagogy can enhance students' understandings of the functions of proof beyond verification. This paper reports on this study and attempts to answer the following research questions:

1) What understanding do students have of the many functions of proof in mathematics?
2) Does exposure to an inquiry pedagogy impact these understandings?

2. Functions of proof

While I will not go into answering the philosophical question of "What constitutes a proof?" in this paper, many definitions of proof put forth by mathematicians and mathematics educators alike focus on the verification power of proof. For instance, the mathematician Krantz (2007) defines proof as "a rhetorical device for convincing another mathematician that a given statement (the theorem) is true" (pg. 3). This reflects a common adage, taken from Mason, Burton and Stacey (1982), that one should develop an argument by first convincing yourself, then by convincing a friend and finally by convincing an enemy. These definitions of proof make clear that the primary, and per- haps only, function of proof is that of verifying that a given statement is true and then convincing others of your result.

A careful analysis of the functions of proof present in teaching and research paints a different picture. A literature review exposes the multitude of roles that proof can play in mathematics beyond merely verification of a statement. In fact, de Villiers (2012) cautions that defining proof as having purpose only for verification of statements can obscure other functions of proof that are perhaps more fruitful in a pedagogical context. I present an outline of eight other functions of proof from the mathematics education literature. These additional functions of proof will be used as a framework for our analysis undertaken to answer the two research questions.

Hanna (1990) and de Villiers (1990) demonstrate that some proofs can be *explanatory* and provide insight as to why a certain statement is true rather than just show that it is true. Hanna (1990) uses the example of two proofs of the statement "*The sum of the first n positive integers, S(n), is equal to* $\frac{n(n+1)}{2}$". A proof of this statement by mathematical induction may give insight into why this formula works for all natural numbers *n*, but gives no clarification about how the formula $\frac{n(n+1)}{2}$ was obtained. On the other hand, Hanna (1990) gives the proof in Fig. 1 as one that proves the statement and also gives an explanation of how this formula was obtained, which she terms a *proof that explains*.

Beyond proof as explanation, de Villiers (1990) presents three additional functions of proof in mathematics. The first is the use of proof as a *means of discovery* since proof can sometimes lead to new results in a field. For example, proofs may be analyzed resulting in a conjecture of a more general case. Indeed, Lakatos (1976) shows that the method of proof analysis, where, upon emergence of a counterexample, proofs and definitions are reanalyzed, is exactly the way that mathematics progressed throughout history and was a catalyst for the emergence of new fields. A second function of proof identified by deVilliers is that of an *intellectual challenge* to the prover. The completion of a proof can be very satisfying for mathematicians and analogous to solving a puzzle. Third, proof can expose logical relationships between statements and can serve as a tool for *axiomatizing results* in a mathematical system. If another statement is used in a proof, we can consider that an axiom or lemma needed for that result and, comparatively, if a further statement is the consequence of that proof we consider it a corollary. This provides a way to determine a hierarchy of statements in mathematics.

One reason that proofs are of the utmost importance in mathematics is that they, instead of axioms and theorems, are the main vehicles in which mathematical knowledge is contained and transferred (Hanna and Barbeau, 2011; Rav, 1999). Thus, proof serves an important *communication* role. Mathematicians talk to each other in the language of proof expressed in publications and presentations. Furthermore, Weber (2002) claims that proof can serve as the *justification for a definition* in mathematics. For instance, Weber (2002) cites the use of Peano arithmetic axioms to prove simple statements such as $2 + 2 = 4$. It is unlikely that this would be proven for verification purposes since it is a simple statement that one is already convinced by, but a proof would instead aim to determine if the axioms of Peano arithmetic are reasonable ones. Weber (2002) also posits that proof can *illustrate techniques* in advanced mathematics.

The author gives the example of instructors asking students to prove that $f(x) = x^2$ is a continuous function. Most students already have an intuitive understanding of why this statement should be true, however the construction of a proof this statement can demonstrate certain techniques for proving statements about continuous functions.

Finally, Yackel and Cobb (1996) comment that another role of proof can be to *provide autonomy* to students by allowing them to verify statements for themselves and, in some cases, to create their own mathematical knowledge. This can be important as it gives

A proof that explains.

$$\text{Let} \quad S(n) = 1 + \quad 2 \quad + \cdots + n$$
$$\text{Then, } S(n) = n + (n-1) + \cdots + 1$$

$$\text{So,} \quad 2S(n) = (n+1) + (n+1) + \cdots + (n+1) = n(n+1)$$
$$\text{Thus, } S(n) = \frac{n(n+1)}{2}$$

Fig. 1. A proof of the statement "The sum of the first n positive integers, S(n), is equal $\frac{n(n+1)}{2}$" that explains (Hanna, 1990, pg. 10).