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Mediational activities in a dynamic geometry environment and teachers' specialized content knowledge☆

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ABSTRACT

Dynamic geometry environments can support learning of geometry through mediating learners' activity. To understand how dynamic geometry environments mediate the activity of mathematics teachers, we used Rabardel's categories of tool mediations in an instrument-mediated activity. We analyzed the discursive and inscriptive interactions of 4 mathematics teachers who worked for 15 weeks as a team to construct geometric figures and solve open-ended geometric problems in a collaborative, dynamic geometry environment. Teachers' specialized content knowledge was evident when they used the environment epistemically. In addition to Rabardel's epistemic and pragmatic mediations, we found and coined a third mediation—pedagogic mediation—by which teachers use the environment to help other team members understand particular geometric objects and relations among them. Understanding how teachers use technological tools can inform the design of professional development programs that engage them with such tools to extend their specialized content knowledge.

1. Introduction

Advances in technology, specifically, digital technologies, influence how people interact with their environment and with each other. In particular, some digital technologies developed for teaching and learning mathematics enable learners to visualize and explore relations among mathematical objects and ideas. Moreover, some technologies afford learners opportunities to interact with each other and collaborate to build mathematical knowledge. To use these technological tools in mathematics classrooms successfully requires understanding of how they influence teachers' and students' social interactions and shape how they build mathematical knowledge. Vygotsky (1978) discusses the role of tools and signs for cognitive development, illustrating how intellectual development occurs through individuals' social engagement in tool-mediated activities. In schools, learners build mathematical knowledge through interacting with classmates and teachers and engaging in activities that are mediated by tools such as dynamic geometry software. To understand how such mathematical activity contributes to how individuals build their mathematical knowledge, careful investigations are needed to interpret how learners interact with mathematical tools and with each other.

Several studies use Vygotsky's notion of mediation to explain how learners interact with technological tools in mathematical activities (for example, Barcelos, Batista, & Passerino, 2011; Hoyles & Noss, 2009; Laborde, 2007; Mariotti, 2000). Technological tools mediate learners' activity and provide additional tools and signs that can support their mathematical discourse and building of

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meaning. Researchers have theorized how tools mediate users' activity while they interact to build meaning and engage in discourse to solve shared tasks (Lonchamp, 2012; Rabardel & Beguin, 2005). With a dynamic geometry environment, other researchers investigate the co-action relations between learners and the environment, specifically, how learners react to an environment's feedback to their actions (Hegedus & Moreno-Armella, 2010; Moreno-Armella & Hegedus, 2009). The environment's feedback is programmed based on the theory of geometry to react to users' manipulations of geometric objects. Such an environment provides new objects and actions that, when internalized, influence learners' mathematical discourse and knowledge construction (Alqahtani & Powell, 2017; Bussi & Mariotti, 2008; Mariotti, 2000; Sinclair & Yurita, 2008). These studies provide insights into how tools influence mathematical learning, which, in turn, allows mathematics teachers to gain an understanding of how tools can be used in their classrooms.

However, to implement mathematical tools effectively in classrooms, teachers need to know how to use the tools for doing mathematics. Their understanding of how to use the tools and how tools shape mathematical discourse and ideas can be seen as part of their technological pedagogical content knowledge (TPACK; Koehler & Mishra, 2008; Mishra & Koehler, 2006) and their mathematical knowledge for teaching (MKT; Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008), which influences students' achievement (Ball, Hill, & Bass, 2005; Hill, Rowan, & Ball, 2005; Rowan, Chiang, & Miller, 1997). This creates a need to understand how teachers use digital technological tools to engage mathematically. Specifically, there is a need to understand how these tools mediate teachers' activities and how their knowledge evolves. These understandings are important for two reasons: They provide insights into how teachers may use digital technological tools in their classrooms and into how such tools may influence elementary, middle, and high school students' mathematics learning.

To understand how technological digital tools mediate teachers' mathematical activity and shape their mathematical knowledge, we investigated mediational activities in a collaborative, dynamic geometry environment, called Virtual Math Teams with GeoGebra (VMTwG),[1] as teams of mathematics teachers worked on mathematical tasks. In VMTwG, users communicate through a chat panel and share a multi-user version of GeoGebra, a dynamic geometry environment, in which they can jointly construct and manipulate geometric objects. Our goal was to understand how teachers interact with technological tools while solving mathematical tasks that promote productive mathematical discourse (Powell & Alqahtani, 2015) and how their interactions shape their mathematical knowledge. This study aims to answer the following questions:

1. How do dynamic geometry environments mediate mathematics teachers collaborative interactions to solve mathematical tasks?
2. How do mediational roles of dynamic geometry environments shape teachers mathematical knowledge?

Here, we present the results of our analysis of the discursive and inscriptive interactions of four middle and high school teachers working as team. We identify different mediations in teachers' activities and the associated mathematical ideas with which teachers engaged. In the following sections, we discuss how previous studies investigated learners' interaction with technological tools and how these studies relate to our theoretical perspective. In particular, we present findings in the literature about how dynamic geometry environments mediate mathematical activity. Then we describe our research methods and results and finally reflect on implications of our results in light of related literature.

2. Related literature and theoretical perspective

To understand how technological tools mediate teachers' activity and shape how they build mathematical knowledge, we draw on the theories of instrumental genesis (Lonchamp, 2012; Rabardel & Beguin, 2005) and mathematical knowledge for teaching (MKT; Ball, Lubienski, & Mewborn, 2001; Ball et al., 2008; Chinnappan & Lawson, 2005; Hill et al., 2008). Instrumental genesis informs our understanding of how VMTwG mediates teachers' activity through analyzing relationships among users, instrument, and objects. We use MKT to investigate how teachers' tool-mediated activity in VMTwG extends their mathematical knowledge. In this section, we discuss instrumental genesis and instrument-mediated activity then discuss how we coordinate it to aspects of MKT.

2.1. Mediational activities in dynamic geometry

Instrumental genesis offers a perspective to understand how a technological environment such as VMTwG, including its dynamic geometry tools, can mediate mathematical activity. It is rooted in Vygotsky's perspective on the role of cultural signs and tools in human development. Based on Marx's theory of historical materialism, Vygotsky believed that material tools developed historically influence human's cognitive development and behavior and further extended the idea to include the role of signs (for example, written and spoken language, number systems; Vygotsky, 1978). Tools and signs mediate human activity differently. Tools are externally oriented while signs are internally oriented. Externally oriented tools guide individual's actions on an object and results in changing the object. In contrast, internally oriented signs are psychological tools that guide cognitive activity and do not aim to change the object directly. Signs are social means that are used to communicate with others before becoming psychological tools that influence one's cognitive activity (Wertsch, 1985). Even though tools and signs seem to account for most aspects of mediated activity,

[1] The environment, Virtual Math Teams (VMT), has been the focus of years of development by a team led by Gerry Stahl, Drexel University, and Stephen Weimar, The Math Forum at the National Council of Teachers of Mathematics, and the target of considerable research (see, for example, Stahl, 2008; Stahl, 2009). Recent research has been conducted on an updated environment, VMTwG, that encompasses a multiuser version of the dynamic geometry environment, GeoGebra, (Alqahtani & Powell, 2015; Grisi-Dicker, Powell, Silverman, & Fetter, 2012; Powell, Grisi-Dicker, & Alqahtani, 2013; Stahl, 2013, 2015).