Journal of Mathematical Behavior 48 (2017) 112–136

Contents lists available at ScienceDirect

Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Pre-service elementary teachers' strategies for tiling and relating area units

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ARTICLE INFO

Keywords:
Pre-service elementary teachers
Area measurement
Enumeration
Strategies
Tiling

ABSTRACT

It has been established that preservice elementary school teachers (PSTs) often employ procedural methods when solving measurement problems without conceptual understanding or flexibility, but a significant gap exists in the literature identifying why. Through the lens of discrete and continuous interpretations of area, this study extends the research base by describing strategies PSTs use to tile a two-dimensional space with varying size tiles and what these strategies imply about PSTs' conceptions of area measurement. These strategies and implied conceptions enable further discussion on the multiple purposes of the area model as an illustrative measure for mathematics throughout the elementary school curriculum.

1. Introduction

Studies of preservice teachers' (PSTs') understanding of mathematics content have consistently shown that the knowledge PSTs bring to their preparation programs and, in some cases, develop within those programs, is largely procedural and insufficient for teaching in K-12 settings. The fragility of PST's understanding is often revealed in their attempts to explain and represent the concepts behind common algorithms and formulas they memorize and use (e.g. Ball, 1988; Ma, 1999; Menon, 1998; Thanheiser, 2010; Yang, Reys, & Reys, 2009). Many PSTs learned topics like addition, subtraction, multiplication, and division superficially, relying on step-by-step procedures (Killpatrick, Swafford, & Findell, 2001) that can be counted on to yield correct answers, but do not, in and of themselves, create the strong and connected understanding of mathematics needed for teaching.

Similarly, when considering geometry and measurement specifically, many PSTs, as elementary students, learned how to calculate and convert measurements by memorizing and applying rules using step-by-step procedures and without attending to underlying concepts (Battista, 2007; Baturo & Nasson, 1996). They demonstrate a procedural understanding of area often limited to memorized formulas disconnected from real-world applications (Baturo & Nasson, 1996; Menon, 1998; Reinke, 1997). To highlight a PST's understanding, consider the example below (Fig. 1) that shows a PST's response when asked how she might verbally and pictorially explain the number of square inches in a square foot.

In the written response, the PST reasoned that there are 12 square inches in one square foot, perhaps assuming that if one foot is equivalent to 12 inches that one square foot is equivalent to 12 square inches. She then explained that a 4-inch by 3-inch rectangle would represent the square foot or 12, referencing area as length multiplied by width. There are several things that are striking about her response. First, she is not consistent in her language and interchanges units freely (i.e. inches and square inches). In addition, she does not attend to the idea that a square foot should be a square with equal side lengths. Lastly, the PST can describe a strategy and relate it to some underlying ideas about area (e.g. area covers a two-dimensional space; finding the area of a rectangle involves

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http://dx.doi.org/10.1016/j.jmathb.2017.05.004
Received 22 June 2016; Received in revised form 26 May 2017; Accepted 28 May 2017
0732-3123/ Published by Elsevier Inc.

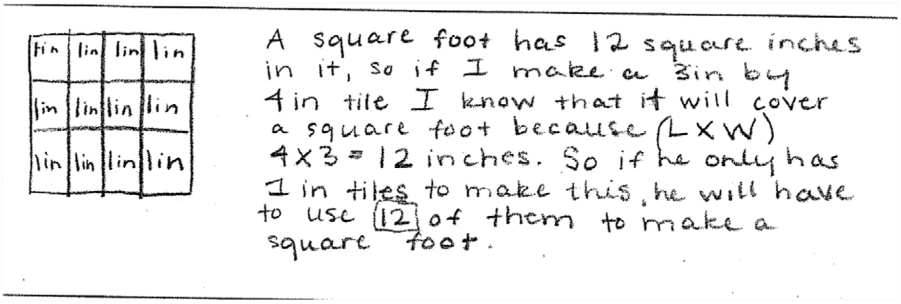

Fig. 1. PST's description of number of square inches contained in square foot.

multiplying length and width), but the answer reveals significant gaps in her understanding of the relation of area units to side lengths. To correctly represent this problem, one must conceptualize a square foot as a square that measures a foot in length by a foot in width or twelve inches in width by twelve inches in length. Twelve square inches fit along the length and fit along the width, so one might conceptualize the number of square inches needed as twelve rows (or columns) of twelve square units.

By applying conversion factors outside of context, many PSTs, like the one highlighted in Fig. 1, enter teacher preparation programs with impoverished notions of measurement (Baturo & Nasson, 1996) and lack a strong sense of number and its relationship to measurement and space (Zevenbergen, 2005). Although PSTs need to understand area measurement concepts for their work as future teachers, research examining how PSTs understand area is disparate and lacks depth. In their review of studies published between 1984 and 2011, Browning, Edson, Kimani, and Aslan-Tutak (2014) found only 26 studies reporting on PSTs' understanding of geometry and measurement, 12 of which focused on area. Furthermore, they found that these studies provide "little depth in either the geometry or measurement content domain" (p. 333). In this paper, we extend the research base by documenting PSTs' strategies for tiling and relating area units. We begin the paper by reviewing related literature.

1.1. PSTs' understanding of area measurement

In reviewing the literature on PSTs' understanding of area, studies center around three themes: 1) understanding the relationship between area and perimeter, 2) knowledge and generation of area formulas, and 3) determining and describing the area of irregular figures. Similar to Ma's (1999) work with in-service teachers, Livy, Muir, and Maher (2012) asked PSTs to compare and contrast the definition of area and perimeter and respond to the claim "I think if the perimeter of a rectangle increases, its area also increases" (p.98). When interviewing 222 PSTs, the researchers found that 72% of PSTs could not provide a correct answer to the claim despite having instructional experiences focused on similar content. In addition, participants tended to define area as "length x width" rather than the space within a region. The researchers noted that it was disturbing that misconceptions remained prevalent even in the final year of college preparation.

Researchers have also investigated PSTs' understanding, use, and generation of area formulas. For example, Baturo and Nason (1996) investigated PSTs conceptions of area in terms of finding the area of multiple and varied region as well as interviewed PSTs on their prior educational experiences with measurement. They presented two tasks that are related to ones posed in this study. For tasks a and b (see Fig. 2), they asked PSTs to calculate the area of the rectangle and, for task a, they also asked if the area was more or less than a square meter. They attended to whether PSTs used formulae and if they incorporated the correct terminology.

For task a, the researchers noted that many students represented the solution as 128 cm rather than 128 cm^2 and 16 of the 18 participants stated that the area was larger than a square meter because there are 100 centimeters in a meter. For task b, all of the students used the area formula but made computational errors in placing the decimal point when multiplying. When asked to

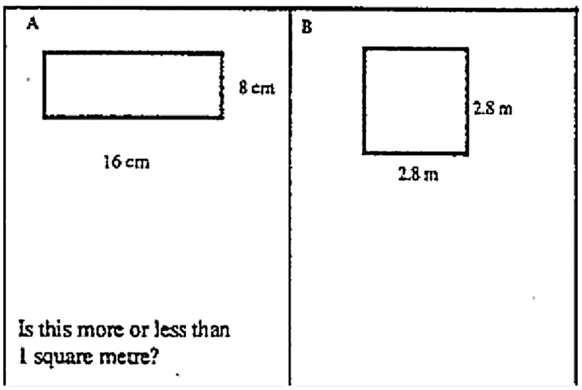

Fig. 2. Rectangular area measurement tasks from Baturo and Nason (1996).