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Engaging students in roles of proof

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ABSTRACT

de Villiers (1990) suggested five roles of proof important in the professional mathematics community that may also serve to meaningfully engage students in learning proof: verification, explanation, systematization, discovery, and communication. We investigate written reflections on an end-of-semester assignment from undergraduates in an inquiry-based transition to proof course, where students reflected on instances during the semester when they engaged in the five roles of proof. We present the types of activities students recalled as influential to their engagement in the roles of proof (presenting, discussing, conjecturing, working on problem sets, and critiquing) and describe how students perceived these activities as influential to their engagement in the roles of proof. We provide student quotations highlighting these activities and offer implications for both research and practice.

1. Introduction

Proof and proving[1] are central to the practice of mathematics. Mathematicians prove to develop and communicate knowledge (Hemmi, 2010; Schoenfeld, 2009), to gain insight and understanding into mathematics or mathematical ways of thinking (Weber, 2010), to make mathematical discoveries (de Villiers, 2012; Komatsu, Tsujiyama, & Sakamaki, 2014), to organize or systematize a body of established knowledge (de Villiers, 1990; Freudenthal, 1973), and to convince oneself or another that a theorem is true (Hersh, 1993; Weber, Inglis, & Mejia-Ramos, 2014). Despite educational recommendations that students learn to prove (Association of Mathematics Teacher Educators, 2017; Common Core State Standards Initiative, 2010; National Council of Teachers of Mathematics, 2000), proof is still largely absent from K-16 classrooms (Stylianou et al., 2009Stylianou, Blanton, & Knuth, 2009). Even university mathematics majors report varied, and often limited, experiences in their exposure to proof at the undergraduate level (Boyle, Bleiler, Yee, & Ko, 2015; Stylianou, Blanton, & Rotou, 2015) and have difficulty constructing and validating proofs (Bleiler, Thompson, & Krajčevski, 2014; Selden & Selden, 2003; Weber, 2001).

While proof serves a variety of important functions for mathematicians, students often do not understand the purpose of proof (de Villiers, 1990). de Villiers (1990) argued that for students to learn proof in a meaningful way, they should understand and experience the various functionalities of proof. In particular, he suggested five roles of proof important in the professional mathematics community that may also serve to meaningfully engage students in learning proof: verification, explanation, systematization, discovery, and communication. Although verification (i.e., obtaining conviction that a mathematical statement is true) is arguably the primary function of proof in the discipline (Hersh, 1993; Staples, Bartlo, & Thanheiser, 2012), this role is often experienced superficially by students as they are asked to prove intuitive results (Schoenfeld, 1994) or instructor-selected theorems (Hersh, 1993). Furthermore,

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[1] In this paper, we follow de Villiers (1990) in defining proof as 'logical deduction' that is used to verify, explain, systematize, discover, and communicate mathematics. Moreover, by proof, we mean to encapsulate both the artifact of proof (noun; a written deductive argument) and the act of proving (verb; the processes involved in deriving a deductive argument) (Stylianides, 2007).

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mathematicians and students often obtain conviction that a mathematical statement is true by consideration of examples (de Villiers, 2004a; Weber et al., 2014). As de Villiers (1990) noted, "conviction is not a bijective function of proof" (p. 23).

While de Villiers and others (Hemmi, 2010; Hersh, 1993) have suggested the importance of students' experiencing proof in ways that extend beyond the verification role, few studies (e.g., de Villiers, 2004b) exist that explore how learning experiences can be structured so that students meaningfully engage in multiple functions of proof. In this study, we look at student reflections from one introduction-to-proof course in order to understand the activities that were meaningful to students' engagement in the roles of proof and how the structure of the space of learning engaged students in those roles. We believe that identifying such activities, informed by student perspectives, could be a productive springboard for identifying interventions that could be further researched to understand their effect on students' engagement with and understanding of proof.

2. Theoretical perspective

The situative perspective (Greeno, 1997; Lave & Wenger, 1991; Wenger, 1998) has been a useful framework for making sense of the teaching and learning of mathematics (e.g. Boaler, 2000; Hemmi, 2010). Under this perspective, learning is viewed as moving "toward full participation in the sociocultural practices of a community" (Lave and Wenger, 1991Lave & Wenger, 1991). As Greeno (1997) asserted, a critical question to consider under the situative perspective is, "Which combinations and sequences of learning activities will prepare students best for the kinds of participation in social practices that we value most?" (p. 9). From this perspective, in order to gauge mathematics learning, one should extend beyond a study of how learners internalize content knowledge, and also consider (a) how learners engage in the practices of the discipline that are valued by the participants of the mathematics community of practice (Wenger, 1998), and (b) the opportunities that open up a space for learners to meaningfully engage in those practices (Greeno, 1997). We look to de Villiers (1990) and his five roles of proof in order to frame our understanding of the practices of the discipline that we value related to proof. Then, we look to Marton, Runesson, and Tsui (2004) in order to frame how we investigate the opportunities that open up a space for engaging in these mathematical practices.

2.1. Roles of proof

The five roles of proof discussed by de Villiers (1990) provide us insight into the practices that are critical to the work of mathematicians and that help delineate important social processes that are sub-components of the practice of proving.[2] In his 1990 article, he described how mathematicians experience the functionality of those five roles in their work. In this paper, we use the phrase *engage in roles of proof* to be consistent with de Villiers' "experiencing the functionality" (p. 23) of proof. We summarize the five roles using this language.

A mathematician engages in **verification** when a proof serves to convince or justify, for the mathematician (or her peer), the truth of a mathematical claim. The reason why a mathematical statement is true may be illuminated as a mathematician engages in the **explanation** role of proof. Mathematicians engage in **systematization** when they use proof to organize and create a deductive system of axioms, definitions, and theorems. A mathematician engaged in **discovery** may deduce an unanticipated result during the completion of a proof. Proof also provides a means for mathematicians to engage in **communication** as they transmit mathematical knowledge to others in their community. We consider each of these roles as corresponding to a specific social practice subsumed under the broader social practice of proving. For example, *using a proof to communicate* or *using a proof to discover* are distinct, but perhaps not exclusive, practices related to proof. Hemmi (2010) remarked, "Experiencing [the roles of proof] could enhance students' feeling of meaning and thus enhance their participation in mathematical practice" (Hemmi, 2010). Hemmi's idea provokes us to look at engagement in the roles of proof from the student perspective, using their reflections to identify activities that enhanced their participation in the mathematical practice of proving.

2.2. Space of learning

Of central importance in this study is understanding "what actually comes to the fore of [students'] attention" (Marton et al., 2004) with respect to the five roles of proof. We follow Marton et al. in their attention to the *space of learning*. We are interested in identifying "effective ways of arranging for learning" (p. 3) that lead students to engage in the roles of proof. In particular, we attend to the perspectives of students as a way to gauge the activities that were most meaningful to them with respect to their learning. Hiebert et al. (1996) discuss *residue* as a way of thinking about what students take with them from classroom experiences, and explain that such residue "might be influenced by the way in which the subject is treated by the curriculum and the teacher, the kinds of tasks students complete, and the everyday rituals of the classroom" (Hiebert et al., 1996). Although Hiebert and colleagues worked from a cognitive perspective and conceived of residue as students' structured internal representations, we reinterpret this concept within the situative lens. What students remember about their participation in classroom practices (i.e., residue) gives insight into activities that were meaningful. In this study, we attempt to understand the elements of the *space of learning* that result in *residue* for students with

[2] Since the publication of de Villiers (1990) work, some researchers, including de Villiers (1999), have considered additional roles. The reader should consult Yopp (2011) for a more comprehensive review of these roles. We limit our study to the five roles introduced by de Villiers (1990) for two reasons. First, de Villiers' (1990) work is highly cited and has had a significant impact on the field of mathematics education (with respect to proof). Second, de Villiers' (1990) paper was accessible to an undergraduate audience and was used to in this study to introduce the students to the roles of proof.