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Reflective abstraction in computational thinking

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ABSTRACT

Computational thinking has become an increasingly popular notion in K-12 and college level education. Although researchers have accepted that abstraction is a central concept in computational thinking, they are quick to disagree on the meaning of it. A focus on reflective abstraction has led to the development of APOS Theory in Mathematics education. This has resulted in many cases of improved student learning in Mathematics (Arnon et al., 2013). Our main aim in this paper is to construct a theoretical bridge between computational thinking and APOS Theory and show that reflective abstraction can be used in the context of computational thinking.

1. Introduction

Wing (2006) made computational thinking popular and proposed that it is not a skill only useful for computer scientists, but rather a fundamental skill that should be learned by everyone. She argued that "to reading, writing and arithmetic, we should add computational thinking to every child's analytical ability" (Wing, 2006, p. 33). Participants of the workshop on The Scope and Nature of Computational Thinking (National Research Council, 2010) considered the question "Why should students learn computational thinking?" They contended that computational thinking is helpful in (i) succeeding in a technological society, (ii) maintaining interest in the information technology profession, (iii) maintaining and enhancing economic competitiveness, (iv) supporting inquiry in other disciplines, e.g. biology, physics, earth sciences and psychology, (v) and enabling personal empowerment. These arguments were influential in a broad spectrum of the academic community and computing courses have become increasingly popular in K-12 education.

Actually the notion of computational thinking is not new to the education community. Although computational thinking has been given significant recognition only recently, it has been present in academic discourse under different forms for decades. Of the numerous definitions for the computational thinking found in the literature we will utilize the definition of Cuny, Snider, and Wing (2010): "the thought process involved in formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information-processing agent" (as cited in Wing, 2011, p. 20).

Despite the popularity of computing courses today, students' difficulties in learning computer science related concepts are well documented in the literature (Boticki, Barisic, Martin, & Drljevic, 2013; Robins, Rountree, & Rountree, 2003). One of the central skills that students should gain in computational thinking is abstraction (Wing, 2006). Piaget developed the notion of reflective abstraction to describe the children's construction of abstract logico-mathematical structures (Beth & Piaget, 1966). He mainly distinguished three types of abstraction: empirical, pseudo-empirical, and reflective abstraction. Our main purpose in this study is to show that reflective abstraction can be used as a tool in the study of computational thinking. For this purpose we will discuss: (i) different views related to abstraction and how Piaget conceptualized reflective abstraction; (ii) APOS Theory as a mathematical learning theory, its

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supporting pedagogy and the role of computational thinking in pedagogy; and (iii) the nature of abstraction and the role of reflective abstraction in computational thinking.

2. Reflective abstraction

In this section we will discuss different views of abstraction in mathematics, describe Piaget's notion of reflective abstraction, discuss the role of reflective abstraction in mathematics and give examples of different views of abstractions.

2.1. Abstraction

The term abstraction means different things to different people and in different contexts. The main meanings we shall consider in this chapter are what we will call, *extraction*, *decontextualization,* and *essence*. The most common, but not necessarily the most important meaning of abstraction of a concept in computer science and mathematics, is extraction, that is, the idea of considering common features of several (the more the merrier) examples and building a structure or category which has all of these features. Thus, these features are all present in all of the examples, but some or all of the examples may also have other features not necessarily possessed by all of the examples. Consider, for instance, all pieces of programming code which have the following features: a control mechanism which is either a variable or is internal and is set at an initial value; variables whose initial values are set; a sequence of lines of code which includes one or more commands that update the control mechanism but may or may not be reached by the flow of control; operations on one or more of the variables that may or may not change their values; a test on the control mechanism; and a transfer mechanism which transfers control out of the sequence of code or returns to the beginning of the sequence of lines of code depending on the result of the test of the counter variable. Such a piece of code is called a *loop* and the abstraction consists of considering any sequence of code having all of the features in the above paragraph to be a loop. Of course, a loop may have other features not shared by all loops, such as printing out the values of some of the variables, changing the values of one or more variables outside of the loop, etc.

The idea of extraction is very close to Piaget's notion of *empirical abstraction*. According to Piaget (Beth & Piaget, 1966), empirical abstraction consists in creating a category by deriving common characteristics from a class of objects. It is largely based on perception, as opposed to reflection. For example, a child may develop the concept of "dog" after seeing many examples, called "dog" by adults, of dogs with four feet, a tail that it wags and a propensity to bark. This conception is good enough to identify animals not previously seen as dogs, although not with perfect accuracy as such a child may go to the zoo and consider that hyenas and lion cubs are dogs. In mathematics, a child may develop a primitive notion of numbers such as "three" by considering many collections of three physical objects and extracting the common property of "threeness". Again, this conception is limited and not very useful for larger numbers or operations on numbers.

Extraction is not a sufficiently powerful mechanism of abstraction even though there is a common belief that most, if not all, abstract mathematical concepts are constructed by means of extraction. An important counter-example is the concept of mathematical group. It is true that there is a vast number of examples of groups in Mathematics and other Sciences. This has led many people to conclude that the concept arose from extracting the four properties in the modern Mathematical definition of group from these examples. But this is not the case. In fact, Galois developed the concept of group, using only one category of examples- the set of permutations of the roots of a polynomial (objects) and composition of two such permutations (binary operation). This strongly suggests that there was something else present in addition to, indeed, before, extraction in the development of the concept of group. For richer conceptions moving beyond extraction, Piaget considered two additional types of abstraction: pseudo-empirical abstraction and reflective abstraction. We will consider these in Section 2.2.

A second meaning ascribed by some to abstraction is decontextualization. Many authors feel that thinking about a concept independently of any context is what makes abstraction difficult (Gravemeijer & Doorman, 1999). Therefore, these authors feel that students should mainly be taught mathematical concepts in some "real-world" context. There are three things that are questionable about such a replacement of decontextualization. First, what is "real-world" will be different for different individuals. You can't use stops on a subway to teach arithmetic to children who have never seen a subway, much less taken a ride on one. Second, there is a real danger that the result of pedagogy focusing on a context will result in the students perhaps learning something about the context, but little or nothing about the underlying Mathematics. Finally, there does not seem to be much, if any, research results that the use of "real-world" or "realistic" contexts is helpful to students who are trying to learn Mathematics that is not based on "real-world" contexts. We will discuss in the next section how Piaget's concept of reflective abstraction includes decentralization and is an effective alternative to extraction.

2.2. Piaget's concept of reflective abstraction

An alternative to thinking of abstraction as extraction is to emphasize the essence of a concept (more information about our notion of "essence" will be found in Section 4). Piaget considered that an individual's understanding of a concept was constructed by that individual (in a social context) in her or his mind. In many works, Piaget described what he believed was the mental mechanism by which an individual can construct all mathematical concepts, at all levels, that is, make abstractions. He called this mechanism *reflective abstraction*. According to Piaget (Beth & Piaget, 1966, p. 189):

"… reflective abstraction consists in deriving from a system of actions or operations at a lower level, certain characteristics whose