Journal of Mathematical Behavior 45 (2017) 1–14

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Preservice teachers' pictorial strategies for a multistep multiplicative fraction problem

Jae M. Baek[a,*], Megan H. Wickstrom[b], Jennifer M. Tobias[a], Amanda L. Miller[a], Elif Safak[c], Nicole Wessman-Enzinger[d], J. Vince Kirwan[e]

[a] Department of Mathematics, Illinois State University, Normal, IL 61790-4520, United States
[b] Department of Mathematical Sciences, Montana State University, Bozeman, MT 59717-2400, United States
[c] Department of Curriculum, Instruction, and Culture, Florida Gulf Coast University, Fort Myers, FL 33965, United States
[d] School of Education, George Fox University, Newberg, OR 97132, United States
[e] Department of Mathematics, Kennesaw State University, Kennesaw, GA 30144, United States

ARTICLE INFO

Article history:
Received 6 October 2015
Received in revised form 12 October 2016
Accepted 15 October 2016
Available online 12 November 2016

Keywords:
Prospective teachers
Fractions
Representations

ABSTRACT

Previous research has documented that preservice teachers (PSTs) struggle with understanding fraction concepts and operations, and misconceptions often stem from their understanding of the referent whole. This study expands research on PSTs' understanding of wholes by investigating pictorial strategies that 85 PSTs constructed for a multistep fraction task in a multiplicative context. The results show that many PSTs were able to construct valid pictorial strategies, and the strategies were widely diverse with respect to how they made sense of an unknown referent whole of a fraction in multiple steps, how they represented the wholes in their drawings, in which order they did multiple steps, and which type of model they used (area or set). Based on their wide range of pictorial strategies, we discuss potential benefits of PSTs' construction of their own representations for a word problem in developing problem solving skills.

1. Introduction

With the release of the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and higher expectations for new teachers, it is more important than ever for preservice teachers (PST) to make sense of fractions beyond algorithmic operations. To become effective teachers, PSTs will need to understand mathematical content and be ready to support elementary students as they develop understandings of fractions beyond computational procedures, such as developing pictorial representations to represent fractions as well as connecting computational operations to story contexts.

PSTs often view fractions through a lens of numerous misconceptions and procedural rules (Graeber, Tirosh, & Glover, 1989; Simon, 1993). Algorithmic procedures often dominate learners' reasoning and hinder their ability to develop conceptual understandings (Glass, 2004; Mack, 2000; Osana & Royea, 2011). It is also difficult for PSTs to conceptualize fractions and operate on them because reasoning about fractions is often in stark contrast to the procedural methods they were taught as

* Corresponding author at: Department of Mathematics (Campus Box 4520), Illinois State University, Normal, IL 61790-4520, United States.
E-mail address: jbaek@ilstu.edu (J.M. Baek).

http://dx.doi.org/10.1016/j.jmathb.2016.10.005
0732-3123/

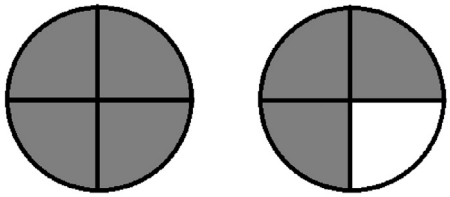

Fig. 1. "7/8 or 1 3/4? " Task to name a fraction that represents the shaded amount (from Tobias, 2013).

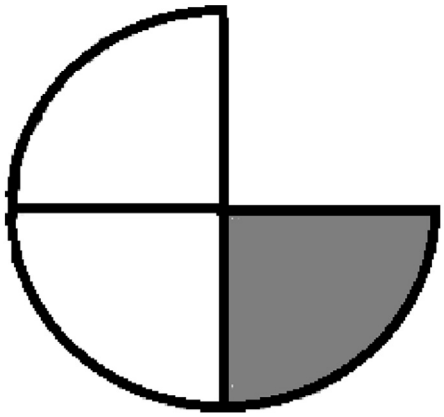

Fig. 2. "1/3 or 1/4? " Task to name a fraction that represents the shaded amount (from Tobias, 2013).

elementary students (Osana & Royea, 2011). Particularly, it has been well documented that many PSTs do not understand the underpinning concepts of fraction operations such as fraction multiplication and division (Ball, 1990; Simon, 1993; Tirosh & Graeber, 1990).

More recently, several studies indicate that PSTs have difficulties with more fundamental concepts of fractions, such as understanding what the referent whole is for a given fraction (Luo, Lo, & Leu, 2011; Tobias, 2013). The studies by Luo, Lo, and Leu (2011) and Tobias (2013) indicate that PSTs need to clearly define the wholes of fractions before they operate on fractions, and a lack of clarity in defining wholes may be related to PSTs' confusion with fraction operations.

Although PSTs exhibit difficulties with fractions, multiple researchers highlighted that this is not always the case with elementary students (Mack 2001; Olive, 1999). Olive (1999) and Mack (2001) investigated how children utilize their knowledge of whole numbers, partitioning, and units, and reported that children could solve fraction problems in a multiplicative context in a way that makes sense to them and explain their method to others. In this study, we extend the research base on PSTs' understanding of referent wholes for fractions by examining the ways in which PSTs define multiple wholes through their valid and invalid pictorial strategies for a multistep word problem.

1.1. PSTs' definition of fractional wholes and its relation to multiplicative computation

Tobias (2013) examined discussions that arose while PSTs solved fraction tasks in which they were required to name a fraction for a shaded portion in a given picture. She documented that PSTs' discussions focused on determining the whole to which their fraction referred and language related to the meaning of the denominator. When Fig. 1 was presented to the class, the PSTs debated whether the shaded portion represented 7/8 or 1 3/4. The PSTs concluded that more clarification was needed when describing the fractions because a fraction that represents the shaded portion in the picture may be 1 3/4 or 7/8 depending on if the referred whole is one circle or two circles.

Tobias (2013) also documented that PSTs realized they needed to reference the whole when discussing a particular fraction. For example, when Fig. 2 was shown, they stated that the fraction could be 1/3 or 1/4. Tobias reported that when PSTs were asked how both could be a possibility, they realized they needed to define the referent whole (i.e., 1/3 of what?) to justify their reasoning.

With regard to operations on fractions, Luo et al. (2011) asked PSTs in the United States and Taiwan to select a pictorial representation that cannot be used to illustrate $3/4 \times 4/5$ or $4/5 \times 3/4$ (see Fig. 3). They found that most participants in both countries had difficulty with this task, and many selected the choice (a) as the incorrect representation. Through a follow-up