Journal of Mathematical Behavior 45 (2017) 35–46

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Teacher attention to number choice in problem posing

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ARTICLE INFO

Article history:
Received 18 December 2015
Received in revised form 2 September 2016
Accepted 3 December 2016
Available online 9 December 2016

ABSTRACT

This study examined how teachers used number choice in contextualized word problems as a pedagogical approach for meeting instructional goals. By collecting and analyzing the contextualized word problems posed by 20 teachers along with their rationales, I identified several means by which teachers used number choice. Additionally, results indicate and characterize a progression for using number choice from no attention to purposeful attention. Implications for decomposing the teaching practice of posing problems to children are discussed.

In the United States, the Common Core Standards for Mathematics (CCSM-M) calls for the solving of contextualized word problems in grades K-6 within several standards. By the end of first grade, for instance, students should solve word problems about a variety of situations, within 20, using objects, drawings, and equations as part of their strategies (CCSSO & NGA, 2010). As students progress through the grade levels, the magnitude of number changes (e.g., within 100 or 1000) as well as the type of strategy (e.g., using objects in kindergarten and strategies based on place-value in second grade) to be used. Left up to the teacher, however, is the design and enactment of pedagogical approaches to progress students in working with higher and more difficult numbers, together with more sophisticated and/or efficient strategies.

This study examined how teachers intentionally used number choice as a pedagogical approach for meeting instructional goals by answering the following research question – "What are the ways in which teachers attend to number choice in their problem posing?" By collecting and analyzing the contextualized word problems posed by 20 teachers along with their rationales, I identified several means by which teachers used number choice. Answers to this question have implications for decomposing (Grossman et al., 2009) the teaching practice of posing problems to children.

1. Study background

First, I discuss two key frameworks in mathematics education around children's mathematical thinking. This research was the basis for the professional development program for which study teachers participated. Second, I briefly examine noticing research and how it contributes to the *professional noticing of children's mathematical thinking* construct. Finally, I highlight research specific to responding to children's mathematical thinking and number choice.

1.1. Children's mathematical thinking

The Cognitively Guided Instruction (CGI) research provided the elementary mathematics field with two significant constructs: 1) problem-type frameworks for addition/subtraction and multiplication/division situations; and 2) frameworks for children's mathematical thinking as children develop numerical concepts (Carpenter, Fennema, Franke, Levi, & Empson,

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http://dx.doi.org/10.1016/j.jmathb.2016.12.001
0732-3123/

1999; Carpenter, Fennema, Franke, Levi, & Empson, 2014). The problem-type framework for addition/subtraction identified eleven different situations, such as joining result-unknown and comparing difference-unknown. A similar problem-type framework was included in the CCSS-M (CCSSO & NGA, 2010), but with twelve problem-types for addition/subtraction.

The frameworks around children's mathematical thinking have been used by teachers and studied by researchers for their effectiveness in supporting responsive teaching practices. These frameworks were constructed on the tenet that "children bring to school informal or intuitive knowledge of mathematics that can serve as the basis for developing much of the formal mathematics of the primary school curriculum" (Carpenter, Fennema, Franke, 1996, p. 6). Further, the frameworks provide guidance in the development of children's mathematical thinking over time. Land and Drake (2014) identified this development as a type of learning progression, and specifically, a student solution progression. Children begin solving problems by direct modeling and progress to more sophisticated strategies such as counting and the use of derived facts (Carpenter, Fennema, & Franke, 1996; Carpenter et al., 2014).

CGI researchers discovered that teachers who used knowledge of children's mathematical thinking provided more problem-solving opportunities, encouraged the use of a variety of strategies, and elicited children's mathematical thinking more than control-group teachers (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). In a longitudinal study, Fennema, Carpenter, Franke, Levi, & Jacobs (1996) found that teachers' beliefs and instruction changed from demonstrating procedures to providing problem-solving situations and opportunities to talk about mathematical thinking, and class achievement increased based on these changes. To further the discussion around teachers' capabilities, I turned to the work around professional noticing of children's mathematical thinking.

1.2. Noticing children's mathematical thinking

In describing the *Discipline of Noticing*, Mason (2002) wrote that professionals notice particular aspects in their professional setting. To "develop professionally requires two things: to increase sensitivity to notice opportunities to act, while at the same time, to have come to mind in the moment when they are relevant, a range of possible appropriate actions" (p. xi). Teaching requires noticing an opportunity for advancing children's understandings; in the complexity of a classroom, it takes expertise to do so. van Es and Sherin (2002) introduced the *Learning to Notice Framework*. This framework outlined three main aspects: "1) identifying what is important in a teaching situation; 2) using what one knows about the context to reason about a situation, and 3) making connections between specific events and broader principles of teaching and learning" (p. 573). The work of van Es and Sherin (2002) decomposed (Grossman et al., 2009) the noticing practice into three skills, allowing the field to distinguish between the skills and focus on teachers' development.

Building on the noticing research, Jacobs, Lamb, & Philipp (2010) proposed the *professional noticing of children's mathematical thinking* construct. Three interrelated skills comprised the construct: *attending* to children's strategies, *interpreting* children's understandings, and deciding how to *respond* on the basis of children's understandings. Jacobs et al's construct (2010) was similar to the one proposed by van Es and Sherin (2002) in that it decomposed the ability to notice into interrelated skills and included identifying what information is important in a situation and reasoning about that information. The Jacobs et al. (2010) construct was different in that it was specific to children's mathematical thinking and included the action step of *responding*.

The noticing skills (attending, interpreting, and responding) have been researched individually or as a set in several studies with in-service or pre-service teachers (PSTs). For example, Star and Strickland (2008) investigated the impact of video viewing on the attending skill. By having PSTs watch a video and answer questions pertaining to what they noticed in the video as a pre- and post-assessment, Star and Strickland (2008) realized that a secondary mathematics methods course had significant impact on PSTs' ability to notice. Working with seven in-service teachers in a video club, van Es and Sherin (2002) obtained two key results: 1) teachers made shifts in their noticing skills in terms of "who and what they found noteworthy, how they analyzed these interactions, and their level of specificity" (p. 253); and 2) teachers developed their noticing abilities in different ways. Barnhart and van Es (2015) studied the relationship between the three noticing skills among PSTs. The researchers found that sophisticated interpretations and responses depended on high levels of attending and that high levels of attending did not always result in sophisticated interpretations and responses. In sum, these results conveyed that the noticing skills could be developed, teachers developed the noticing skills in different ways, and the noticing skills were interrelated.

In addition to the above results, Jacobs et al. (2010) identified a list of growth indicators that detailed shifts in teachers' noticing abilities. Evidence of these growth indicators appeared in participants' problem posing responses to children's mathematical thinking in the form of video or written work. Because this study focused on problem posing with attention to number choice, I have listed the growth indicators related to responding only.

- A shift from considering children only as a group to considering individual children, both in terms of their understandings and what follow-up problems will extend those understandings.
- A shift from reasoning about next steps in the abstract (e.g., considering what might come next in the curriculum) to reasoning that includes considerations of children's existing understandings and anticipation of their future strategies.
- A shift from providing suggestions for next problems that are general (e.g., practice problems or harder problems) to specific problems with careful attention to number selection. (Jacobs et al., 2010; p. 196).