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The effect of rewording and dyadic interaction on realistic reasoning in solving word problems

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ABSTRACT

In this study we investigated the effect of the request to reword the text of problematic word problems on the occurrence of realistic answers. We proposed the activity of rewording four word problems to fifth-grade pupils either working individually or in dyads. We found that for one of the problems the rewording in dyads produced a strong increase in pupils' realistic answers, for another one a small increase, and no increase at all for the other two. We also analysed pupils' reworded texts in order to characterize the kind of information added. For all four problems the rewording in dyads produced richer texts in comparison to the individual rewording. Moreover, for the problems where an increase in realistic answers was observed, two particular kinds of information, named descriptive and action information, were added by dyads who answered realistically.

1. Introduction

The well-known *l'age du capitaine* study (Baruk, 1985) was one of the most popular studies that brought to the attention of the research community the phenomenon known as 'suspension of sense-making' when solving word problems (Schoenfeld, 1991). One version of the problem is the following one "On a boat there are 26 sheep and 10 goats. What is the age of the captain?" (Baruk, 1985). All over the world, it is found that the more pupils become proficient in adding two-digit numbers, the higher becomes the proportion of pupils who give a response to the problem by adding the two numbers (e.g., Merseth, 1993). In other words, the more pupils advance in their school path, the more we can witness a tendency to unthinkingly apply arithmetic operations when responding to word problems, without critically considering the reality that the word problem is actually referring to. This finding is very problematic, given that one of the major justifications for the important role of mathematics in primary and secondary school curricula and for including word problems in the mathematics class is exactly that mathematics provides a set of tools for understanding real world situations (e.g., Blum & Niss, 1991).

Explanations that have been raised for pupils' tendency to ignore real-world considerations often refer to the stereotypical nature of the word problems typically used in school, in the sense that they can be solved straightforwardly and unquestionably by applying the most obvious arithmetic operation(s) with the given numbers, while, as is well known, mathematical modelling from reality is far from straightforward and linear. The implicit and explicit rules which govern the

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educational practices surrounding such word problems have also been called the "didactical contract" (Brousseau, 1986). This theoretical construct describes some of pupils' mathematical behaviours in classrooms in the light of the reciprocal tacit expectations and obligations that guide the relationship between learners and teachers (Brousseau, 1980; Education Committee of the EMS, 2011). In a mathematics education context, where the choice of the suitable intellectual behaviour is not straightforward (sometimes pupils need to remember, other times it is more important that they explore or think productively, etc.), the risk is that pupils (especially the unsecure ones) think not about "what is suitable to do" but about "what the teacher wants them to do". In this sense the activity of solving word problems is often perceived by pupils as an occasion to exhibit their ability to perform arithmetical operations quickly and accurately, rather than an occasion to engage in mathematical modelling.

The captain's age problem by Baruk (1985) is just one manifestation of a larger problem. During the last decades many studies have been reporting pupils' difficulties in treating word problem solving lessons as experiences of mathematical modelling (Greer, 1993; Verschaffel et al., 1994). In the attempt to deepen the understanding of this phenomenon, a particular line of studies has started to work with a particular kind of non-routine word problems, so-called problematic items (P-items) – where peculiarities of the everyday life situation described in the word problem need to be taken into account in order to give a meaningful answer. An example of P-item is the balloons item: *Grandfather gives his 4 grandchildren a box containing 18 balloons, which they share equally. How many balloons does each grandchild get?* When pupils then give a response to such P-items without considering in some way the peculiarities of the everyday life situation, this may indicate that pupils are unable or not inclined to consider the word problems as exercises in mathematical modelling (Van Dooren, Verschaffel, Greer, & De Bock, 2006).

In the study reported in this paper, we investigated the effect of inviting pupils to reword and elaborate four given P-items–individually or in dyads–on the realistic nature of their answer to those problems. The hypothesis was that the request to elaborate the P-items texts would lead the pupils to build richer situation models in comparison to the one built when merely solving the P-items. In this sense the building of richer situation models can support pupils in taking into consideration the constraints and peculiarities of the real-world contexts of these P-items.

Moreover, we analysed the information added in pupils' reworded problems to get a deeper understanding of the model they built of the situation, and to investigate whether there is a relationship between pupils' situation models and the realistic nature of their answers in these P-items.

2. Theoretical and empirical background

A word problem can be defined as:

> "[. . .] a text (typically containing quantitative information) that describes a situation assumed as familiar to the reader and poses a quantitative question, an answer to which can be derived by mathematical operations performed on the data provided in the text." (Greer, Verschaffel, & De Corte, 2003, p. 271).

Word problem solving comprises an important aspect of mathematical school life. It consists of complex processes in which various levels of representation occur and interact: from the process of initial interpretation of the text base (see e.g., Kintsch & van Dijk, 1978; Kintsch, 1988), to the building of, what many scholars have called, a *situation model* (see e.g., Kintsch & Greeno, 1985; Kintsch, 1998; Van Dijk & Kintsch, 1983) leading to the construction of a proper mathematical model and its solution by means of the suitable computations (see e.g., Reusser, 1990; Verschaffel et al., 2000). Reusser (1990) described the situation model as an intermediate and less mathematical representation constructed by using one's real-world knowledge and personal experiences to interpret the information found in the text base. In text comprehension literature, many deep reflections and tools of analysis have been developed in the attempt to catch the complexity of the knot between the initial text interpretation and the building of the situation model (see e.g, Zwaan & Radvansky, 1988). Given that we focus on mathematical word problems, our attention mainly goes to another representational transition, namely from the building of the situation models to the construction of a mathematical model (see e.g., Reusser, 1990; Verschaffel et al., 2000). Indeed one of the main goals of word problems is to bring pieces of reality in the classroom in order to let pupils experience different aspects of mathematical modelling and problem solving processes, without the practical inconvenience to make a direct contact with real world contexts (Verschaffel et al., 2000). This application of mathematics to real world problems is a complex process involving a number of phases. While there are many characterisations of the process of mathematical modelling – and thus also of word problem solving – (e.g., Blum & Niss, 1991; Burkhardt, 1994; Verschaffel et al., 2000), they essentially all involve the following components: understanding and defining the problem situation leading to a situation model; constructing a mathematical model of the relevant elements, relations and conditions embedded in the situation; working through the mathematical model to derive some mathematical results; interpreting the outcome of the computational work in relation to the original problem situation; evaluating the model and the interpreted mathematical outcome; and communicating the obtained solution of the original real-world problem. This modelling process, as illustrated in Fig. 1, is typically not followed in a strictly linear way, but rather cyclically (Blum & Niss 1991; Burkhardt 1994; Verschaffel et al., 2000).

In the 90′s, two studies (Greer, 1993; Verschaffel et al., 1994) provided further empirical evidence that the goal of word problems to bring mathematical modelling experiences into the classroom is very often not met. In these studies, it was found that upper elementary pupils only very rarely make realistic considerations when solving word problems. This was