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Authority and whole-class proving in high school geometry: The case of Ms. Finley

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ABSTRACT

Students' experiences with proving in schools often lead them to see proof as a static product rather than a negotiated process that can help students justify and make sense of mathematical ideas. We investigated how authority manifested in whole-class proving episodes within Ms. Finley's high school geometry classroom. We designed a coding scheme that helped us identify the proving actions and interactions that occurred during whole-class proving and how Ms. Finley and her students contributed to those processes. By considering the authority over *proof initiation*, *proof construction*, and *proof validation*, the episodes illustrate how whole-class proving interactions might relate to students' potential development (or maintenance) of authoritative proof schemes. In particular, the authority of the teacher and textbook limited students' opportunities to engage collectively in proving and sometimes allowed invalid arguments to be accepted in the public discourse. We offer suggestions for research and practice with respect to authority and proof instruction.

Introduction

Students across grade levels have demonstrated difficulty constructing and validating mathematical arguments (Healy & Hoyles, 2000; Hsieh et al., 2011). Any such widespread difficulty is certain to have multiple contributing factors, such as curricular resources (Fujita & Jones, 2014), teachers' knowledge (Bleiler, Thompson, & Krajcevski, 2014), and instruction (Bieda, 2010). With regard to instruction, when proof is "done to students" (Harel & Rabin, 2010)—that is, when students are taught the procedural rules of proof but not the intellectual motivations behind them—students may develop an authoritative proof scheme (Harel & Sowder, 2007) and may see proof as a static product (Furinghetti & Morselli, 2011) that is valid based on some external authority. This view runs counter to the notion of proof as a negotiated process that students can engage with to make sense of mathematics (Boyle, Bleiler, Yee, & Ko, 2015; Staples, Bartlo, & Thanheiser, 2012) and build shared understanding (Bleiler, Ko, Yee, & Boyle, 2015; Koestler, Felton, Bieda, & Otten, 2013).

In this article, we focus on proof instruction within a single high school geometry classroom. Although we define a *proof* as a valid deductive argument constructed using previously established definitions or propositions that establishes a mathematical claim, and *proving* as the process of attempting to construct a proof, we examined various interactions that were explicitly identified by the class as "proof," whether or not the resulting argument met our definition. We present the case of Ms. Finley, whose proof instruction involved concentrations of authority with the teacher and textbook, shaping students' opportunities to engage collectively in proving and sometimes allowing invalid arguments to be accepted in the public discourse. Although previous research has provided insight

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into the limitations associated with individual students who ascribe to externally-based authority related to proof (summarized in Harel & Sowder, 2007), we do not yet have a clear sense of how objects and people in mathematics classrooms assume authority during proving interactions. Some researchers have identified the teacher's responsibilities in proving interactions (Herbst & Brach, 2006) or investigated proving interactions in particular classroom contexts such as algebra classes (Harel & Rabin, 2010) or an honors geometry class (Martin et al., 2005), but further classroom research is needed because the dynamics of proving can vary substantially across contexts. Thus, we examined whole-class proving episodes in Ms. Finley's non-honors geometry class and focused on authority with respect to proof initiation, proof construction, and proof validation.

2. Literature review

2.1. Theoretical perspective

We view mathematics learning as a process of coming to participate in mathematical discourse (Pimm, 1987). Within any instance of discourse, there are always meanings being construed, discourse patterns being enacted, and interpersonal relationships being negotiated (Halliday and Matthiessen, 2003). One particularly important aspect of mathematical discourse is the discourse of proof, both spoken and written. In order to learn to prove, students must have opportunities to participate in this discourse and to work toward fluency in its meanings and patterns. Therefore, we focus on whole-class interactions explicitly marked in the discourse by the word "proof," not because these are the only opportunities for students to engage in the discourse of proof (e.g., they also write proofs individually) but because these are public interactions that yield to inspection. Moreover, while mathematical meanings are being construed and patterns enacted in the whole-class interactions, authority is constantly being negotiated. Pace and Hemmings (2007) define authority as "a social relationship in which some people are granted the legitimacy to lead and others agree to follow" (p. 6). Using this definition, we can interpret a person or object (e.g., the textbook) as having authority over part of a proving interaction if they lead the interaction or direct the behavior of others. Our use of an authority frame differs from those who may focus instead on personal identity or structural power relations (Gutiérrez, 2013). And although there are clear connections (e.g., students developing identities as provers or as someone who dislikes proof; a Western tradition that has the power to determine what will be proven and how), the authority construct aligns well with the scope of the data in this study and the fact that we are analyzing the collective authoring of proofs.

Because students are learning to participate in the mathematical discourse of proving, and teachers and textbooks act as representatives of that official discourse community, teachers and textbooks can be expected to commonly hold substantial authority, and indeed they do (Herbel-Eisenmann, 2009; Muis, 2004). Although their authority is not inherently undesirable, scholars (e.g., Kosko, Rougee, & Herbst, 2014; Lampert, 1990; Webel, 2010) have emphasized the importance of cultivating shared authority in mathematics classrooms. Benne (1970), in a classic piece on anthropogogical authority, wrote about a gradual "movement toward collegiality between teacher and student" (p. 400) wherein the teacher begins as the authority—in our case, the authority on proof—but fosters students' experiences in ways that eventually lead to mutual authority. From our theoretical perspective, shared authority involves students' opportunities to be led and also to lead mathematical discourse, aligning with the goal of full participation in the discourse community.

Regarding whole-class proving in particular, authority issues become complex because authority is negotiated during several phases of interaction—the initiation of a proof (who determines a claim and that it should be proved?), the construction of the proof (who leads the discourse as the argument is articulated?), and the conclusion of the proof (who confirms that the proof is complete and correct?). We refer to these aspects of authority as authority over *proof initiation*, authority over *proof construction*, and authority over *proof validation*, respectively. Teachers, students, and textbooks may all take on authority roles at different times and to different degrees (e.g., Steele & Rogers, 2012), and they may also appeal to rituals or the broader discipline of mathematics as a guide or external arbiter.

2.2. Authority and proving

With regard to textbooks, Otten, Gilbertson, Males, and Clark (2014) found that geometry textbooks in the United States overwhelmingly supply the claims to be proved and regularly specify the format, contents, or outline for proofs, thus exerting authority over proof initiation and construction. Similarly, Fujita and Jones (2014) raised concerns that geometry textbooks in Japan may communicate to students that proofs are arbitrarily required by the educational system or the discipline rather than intellectually necessary when justifying general conjectures.

Yet, teachers and students have control over textbook enactment and so an important question is how they share authority during collective enactment. Herbst and Brach (2006) explored this question and found that students often expect teachers to supply the claims and "givens" of what is to be proved and students only expect to be responsible for completing the proof using recently learned theorems or definitions. These findings relate to what we have called authority over proof initiation (teacher) and proof construction (students). With regard to authority over proof validation, Amit and Fried (2005) studied teachers who told their classes that the students, not just the teacher, must be convinced by mathematical arguments—in our terminology, these teachers were calling for shared authority. Nevertheless, Amit and Fried found that teachers retained "immense authority in the eyes of the students" (p. 145).

Other scholars (e.g., Boyle et al., 2015; McCrone & Martin, 2009) have confirmed the general expectation that teachers spur and confirm proofs, which runs counter to Gravemeijer's (2004) call for students to "only accept new mathematical knowledge of which they can judge the validity themselves" (p. 109). Weber, Inglis, and Mejia-Ramos (2014) on the other hand, has pointed out that it is