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Opportunity to communicate: The coordination between focused and discerned aspects of the object of learning

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ABSTRACT

There are extensive concerns pertaining to the idea that students do not develop sufficient communication abilities in algebra and in mathematics more generally. This problem is at least partially related to their algebraic thinking. Although teaching should give students the opportunity to develop their ability to communicate, there are limited research insights as to why some forms of communication work better than others, and how and why instruction influences such communication. Two case studies are reported on in this article. The analysis of the opportunity to communicate was grounded in variation theory. Differences between focused aspects and discerned aspects of the object of learning are described. The results show that the coordination between the aspects focused on by the teacher and discerned by the students provides students with the opportunity to successfully communicate the content in algebra. In addition, the structure of the lesson influences the opportunity to communicate aspects of the content.

1. Introduction

During the last five years, the educational reform movement in Sweden has advocated increased emphasis on student communication, since many mathematics classrooms remain sites where little to no communication that focuses on the mathematical content occurs. This means that the teacher's main role in the classroom is to help students individually, when they ask for help and that students most of the time are left to work by themselves with their textbook (Johansson, 2006; SOU, 2004:97 The Swedish National Agency for Education, 2008, 2009). Policy and curricular changes reflect this new emphasis: "Teaching should give students the opportunity to communicate using different forms of expression. [. . .] Teaching in mathematics should give students the opportunity to develop their ability to: [. . .] communicate mathematical thinking orally, in writing, and in action" (The Swedish National Agency for Education, 2011, p. 90). In this context some important questions arise: In what ways gives teachers the opportunity to communicate algebra in the classroom in order to help students improve how they communicate in algebra? How does lesson structure (hierarchical or sequential) contribute to improving students' communication abilities?

Recent educational research has stressed the importance of making thinking public (i.e., Bauersfeld, 1995; Fello & Paquette, 2009; Hodge, 2009; Kastberg, Norton, & Klerlein, 2009; Stockero & Van Zoest, 2011; Yackel & Cobb, 1996). In the process of making thinking public, students 1) share their thinking by negotiating the meaning of mathematical ideas with others and 2) defend and justify their reasoning so that they can convince others of the legitimacy of their ideas. In this article the sharing of thinking is used to analyze the aspects focused on/discerned in the communication that occurs in two

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classrooms. *Focused aspects* are aspects of the object of learning that the teacher intends to use in the classroom or aspects that are enacted in the classroom in order to make it possible to discern aspects of the object of learning. The *discerned aspects* refer to aspects that students experience in the classroom.

Although teaching should give students the opportunity to develop their ability to communicate, there are limited research insights as to why some types of communications work better than others, and how and why instruction influences such communication. The focus, in this article, is not on general aspects of communication, but instead on those aspects of the object of learning that the teachers focused on and what aspects the students discerned through the mathematical communication (with a focus on algebra). To do this, a new approach to communication, which has been developed in recent years, is used as a starting point, namely variation theory (Bowden & Marton, 1998; Marton & Booth, 1997; Marton & Tsui, 2004; Marton, 2015; Runesson, 1999). From perspective of variation theory, Olteanu (2014) states that communication is a collectively performed patterned activity in which an aspect that is critical for one or more students (A) is focused on by the action of the teacher or other students (B) so that A discerns the aspects focused on by B. What the teachers focus on (intended or enacted aspects) and what the students discern (experienced aspects) in mathematical communication are aspects of the object of learning (Olteanu & Olteanu, 2013). If these aspects are not distinguished, by experiencing variation, they become *critical aspects*. Critical aspects are those necessary for understanding the content worked out in the classroom. For example, if the content worked on in the classroom is about triangle and rectangle areas, the students should be able to discern that the use of mathematical symbols has different meanings in different mathematical contexts. For instance the formula for calculating the area of a triangle is written as $\frac{1}{2}bh$ and the formula for calculating the area of a rectangle is lxb. In these formulas, b has different meanings, namely the length of the base of the triangle and the breadth of the rectangle. If the students do not understand the use of the symbols, these symbols become critical aspects in the understanding of the formula for calculating the area of a triangle or a rectangle. "Critical" here refers to difference in the learners' ways of grasping and becoming acquainted with the object of learning (Olteanu & Olteanu, 2010).

The object of learning tells us what the students are supposed to become able to do. What the students are expected to become able to do is presented in the educational objectives, but these objectives do not say what the students are expected to learn and implicitly to communicate (Marton, 2015). Marton (2015) specifies that students are expected to learn to simultaneously discern and take certain aspects of the object of learning into consideration. The object of learning is the cornerstone of classroom activities in two main ways. Firstly, the object of learning consists of a sequence of certain aspects presented in a cultural mathematical way, that is, aspects of the object of learning which link to cultural traditions, codes, symbols, and specific ways used to reason and to understand those aspects. Secondly, the object of learning can only be attained through actions directed to discerning certain aspects of this object.

McDonnell (1995) specifies that the concept "opportunity to learn" was originally defined as the overlap between the information students were taught and the information on which they were tested. The definition of the concept has expanded to include the quality of resources, school conditions, curriculum, and teaching that students experience. Marton and Booth (1997) used the concept of opportunity to learn to explain what students might learn in lessons taught with variation. Häggström (2014) shows that how the content is handled in the classroom contributes to a difference in students' opportunities to learn in terms of the dimensions of variation. In this article, the focus is on the *opportunity to communicate* aspects of the object of learning by using concepts from variation theory. The analysis of communication concerns how the content is processed in the classroom because how it is done affects which aspects the students have the opportunity to discern.

2. Theoretical framework

Variation theory (Bowden & Marton, 1998; Marton & Booth, 1997; Marton & Tsui, 2004; Marton, 2015; Runesson, 1999) provides a framework that should make it possible to discern and describe differences in how aspects of the algebra content are communicated, because it has an explicit focus on the object of learning and discernment of its aspects. For discernment to occur, students must experience variation. Experiencing variation in certain aspects require opening up *dimensions of variation* in these aspects. Marton (2015) argues for the difference between *aspects* (the experience of difference) and *features* (the things that differ in an aspect). An aspect is a dimension of variation and a feature is a value in that dimension. In accordance with the distinction between aspect and values, a distinction is made also between critical aspects and critical value.

Marton (2015) has identified four *patterns of variations*, which can facilitate students' discernment of critical features or critical aspects of the object of learning as follows: *contrast, separation, generalization and fusion*. Contrast means that to discern a quality, X, a different quality, non-X, needs to be experienced simultaneously. Generalization means that to discern a value (X1) in one of the dimensions of variation from other values in other dimensions of the variation, the value (X1) needs to remain invariant while the other dimensions vary. Fusion is to experience two (or more) dimensions of variation simultaneously. Olteanu and Olteanu (2011) found a new pattern of variations named *similarity*, which is the property of two or more expressions to adopt the same meaning. A *meaning* is a distinction-based category and "[. . .] denotes 'something particular,' which is marked or indicated, so that it is automatically distinguished from 'something different' or from 'all the rest, which remains unmarked or ignored" (Staude, 2012; p. 8). The similarity also allows for the possibility of discerning different representations of the aspect of the object of learning. Different patterns of variation promote different opportunities for discernment during classroom communication, and some patterns have been found to be effective than others (Holmqvist, 2011). For example, the use of fusion as pattern of variation in the beginning of the lesson does not make