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Overcoming a 4th grader's challenges with working-memory via constructivist-based pedagogy and strategic scaffolds: Tia's solutions to challenging multiplicative tasks

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ABSTRACT

This case study addressed a twofold problem: (a) how a particular, complex scheme that coordinates multiplicative and additive operations may evolve for a student (Tia, grade-4) who was identified as having a learning disability and working memory deficits and (b) how may a constructivist-based approach, Student-Adaptive Teaching (and special considerations of a student's tendency to struggle with working memory) foster the student's construction of such a scheme. The present study consists of qualitative analysis of teaching considerations for and interactions with Tia during episodes in which she was engaged in solving multiplicative reasoning tasks. We discuss three contributions of Tia's case study: (a) the non-trivial mathematics students like her can undertake and enjoy learning, (b) the critical role that analysis of a child's current conceptions serves in adapting goals/activities for her learning of such mathematics, and (c) the potential benefits of utilizing student-adaptive teaching methods for students with learning disabilities.

1. Introduction

Building on research and teaching conducted with normally-achieving students, this study addressed a twofold problem: (a) how particular schemes for reasoning multiplicatively may emerge in a student with a learning disability (LD) in mathematics (specifically, working-memory deficits) and (b) how a constructivist-based, student-adaptive pedagogy can foster students' construction of those schemes. To this end, we have discerned a single case study from a larger project (Xin, Tzur, & Si, 2008) —a constructivist teaching experiment (Cobb & Steffe, 1983; Steffe, Thompson, & von Glasersfeld, 2000) conducted by researcher-teachers who utilized student-adaptive pedagogy to promote multiplicative reasoning in students with LD in mathematics. The larger project targeted the development of educational software to help teach multiplicative reasoning to

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students with LD (Cetintas, Si, Xin, & Hord, 2009; Steffe, 1990; Tzur et al., 2013; Xin et al., 2012; Xin, 2008). In this paper we focus on challenges facing a student identified as having working-memory deficits while she constructs schemes that require coordination of both multiplicative and additive operations with/on composite units (Steffe, 1994; Steffe & Cobb, 1998). This study highlights the interweaving of pedagogical adaptations to fit the student's progression based on her mathematical conceptions and working-memory needs and capacities.

This study is significant in specifying how reasoning needed to support mathematical concepts for meaningfully operating in a place value, base-ten number system may be engendered in students who have often, historically, not been expected or taught to construct the schemes examined in this paper. The continual adaptations made by the researcher-teacher to meet the changing needs and evolving conceptualizations of the student (Steffe, 1990; Tzur, 2007) seem important for both mathematics education and special education (Tzur et al., 2013; Xin et al., 2009). In particular, this case study contributes to understanding of and working with struggling learners to foster their construction of schemes for reasoning multiplicatively that underlie fractional, proportional and, eventually, algebraic reasoning (Confrey et al., 2012; Council of Chief State School Officers and National Governors Association [CCSSO & NGA], 2010; Lesh, Post, & Behr, 1988; National Council of Teachers of Mathematics [NCTM], 2000; Vergnaud, 1983).

This study extends recent work on how teaching rooted in a constructivist theory of learning, with or without computer software, can promote multiplicative reasoning in students identified as having LD (see Tzur et al., 2013; Xin et al., 2009). In particular, this study built on the 6-scheme learning trajectory postulated in regards to how students can undertake the transition from additive to multiplicative reasoning, which Harel and Confrey (1994) and Simon and Blume (1994) identified as a serious challenge for all students. The trajectory is based on research on children's construction of knowledge of composite units: a unit that is made of sub-parts (Steffe, 1994). The project was completed in the context of multiplicative reasoning when students demonstrate understanding of composite units in a variety of contexts such as equal groups situations (e.g., 4 towers of 3 cubes is 12 total cubes) (Fuson & Willis, 1988; Steffe, 1992) and more advanced situations where students recognize the underlying structure of composite units and apply this knowledge to a variety of tasks that increase in complexity (Tzur et al., 2013). In this study, we focused on a 4th grader's (Tia, pseudonym) construction of the 3rd and 4th schemes in the trajectory (see Conceptual Framework), because these two schemes are considered memory demanding due to the multi-step mental processes involved. To construct and use these schemes meaningfully, a student's working memory is challenged to cope with storing/retrieving information from the first and second steps of her solution to a problem while trying to process the third step, and to combine information from all steps to formulate a solution (Swanson & Beebe-Frankenberger, 2004).

In articulating how students with LD are *able* to construct the multiplicative schemes examined in this paper, we contribute to addressing recent mandates for mathematics education to enable such students' success with grade-level content. This expectation is alluded to, for example, in the *No Child Left Behind Act* (NCLB) (2002), in the *Individuals with Disabilities Education Act* (IDEA) (2004), and in recommendations for reform in mathematics teaching (e.g., NCTM, 2000). This expectation is also supported by special education scholars (e.g., Hunt & Vasquez, 2014; Woodward, 2004) who emphasized that students with LD should be provided with key supports, but also be expected to succeed with the general education curriculum and demonstrate proficiency on high-stakes assessments at grade level. Likewise, in the US, the wide adoption of the Common Core State Standards in mathematics (CCSSO & NGA, 2010) increased expectations for learning outcomes to be achieved by such students (Powell, Fuchs, & Fuchs 2013). Specifically, students with LD are expected to meaningfully solve challenging, multi-step problems (see Partnership for Assessment of Readiness for College and Careers [PARCC], 2013). The next section delineates the conceptual framework that underlies our twofold examination of (a) how schemes for reasoning multiplicatively may evolve and support students' problem-solving coupled with (b) how student-adaptive pedagogy may foster this reasoning while addressing challenges to working-memory that multi-step problems present.

2. Conceptual framework

In this section, we describe the strategies and constructs that guided this study. As part of the larger project (Xin, Tzur, & Si, 2008), this study builds on a combination of expertise of researchers from special and mathematics education, along with expertise of researchers in computer science (Cetintas, Si, Xin, & Hord, 2009; Tzur et al., 2013; Xin, 2012). At the heart of our project's work was to acknowledge that the participant may be predisposed to struggle with working memory, but to focus heavily on what a student has and can thus use as a basis for subsequent learning by designing learning situations that target high yet conceptually attainable expectations. We begin with the specific needs of students with LD and corresponding teaching strategies from special education literature, and then delineate the combination of general and content-specific theories and practices that informed our study.

2.1. Teaching students with learning disabilities

Historically, researchers in special education have expressed concerns regarding the various challenges that students with special needs may face when engaging in learning mathematics within constructivist-informed environments (Baxter, Woodward, & Olson, 2001; Woodward, 2006). When teaching is based on constructivist principles of learning, special education and mathematics education researchers have recommended maintaining high expectations, but carefully providing extra supports for students with LD to avoid frustration, while still allowing students opportunities to reason mathemati-