Journal of Mathematical Behavior 44 (2016) 65–75

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Why research on proof-oriented mathematical behavior should attend to the role of particular mathematical content

Paul Christian Dawkins[a,*], Shiv Smith Karunakaran[b]

[a] Northern Illinois University, Department of Mathematical Sciences, Watson Hall 320, DeKalb, IL 60115, United States
[b] Washington State University, Department of Mathematics and Statistics, P.O. Box 643113, Pullman, WA 99164-3113, United States

ARTICLE INFO

Article history:
Received 25 March 2016
Received in revised form 13 October 2016
Accepted 13 October 2016

Keywords:
Proving
Mathematical meanings
Comparative analyses

ABSTRACT

Because proving characterizes much mathematical practice, it continues to be a prominent focus of mathematics education research. Aspects of proving, such as definition use, example use, and logic, act as subdomains for this area of research. To yield content-general claims about these subdomains, studies often downplay or try to control for the influence of particular mathematical content (analysis, algebra, number theory etc.) and students' mathematical meanings for this content. In this paper, we consider the possible negative consequences for mathematics education research of adopting such a content-general characterization of proving behavior. We do so by comparing content-general and content-specific analyses of two proving episodes taken from prior research of the two authors and by re-analyzing the data and results presented in one instance of research from the field. We intend to sensitize the research community to the role particular mathematical content can and should play in research on mathematical proving.

1. Introduction

Since at least the time of Euclid's geometry, proving has been understood to characterize mathematics as a discipline. Inasmuch as mathematics educators endeavor to engage students in authentic mathematical activity, they have expended much effort to provide students with meaningful proving experiences and document the emergence of proving as a mathematical practice among novices. While we certainly endorse this agenda for instruction and research, we are concerned that framing mathematical proving as a single, content-general practice may inappropriately downplay the role particular mathematics content plays therein. We observe two trends in the research literature on mathematical proving: (1) making content-independent claims about mathematical proving using data from a particular mathematical context (i.e. analysis, algebra, number theory, geometry) or (2) eliciting the proving behavior of the same students in multiple mathematical contexts in order to make content-independent claims about proving. In this paper, we consider the possible consequences for research on mathematical proving of downplaying the role of particular mathematical content. We do not intend to deny the validity or value of prior research framed in a content-independent manner (some of which we authored), but rather seek to sensitize the community to possible blind spots induced by common lenses applied to research data and to endorse a research agenda focused on the interplay between proving and particular mathematical content.

* Corresponding author.
E-mail addresses: pdawkins@niu.edu (P.C. Dawkins), shivk@math.wsu.edu (S.S. Karunakaran).

http://dx.doi.org/10.1016/j.jmathb.2016.10.003
0732-3123/

Furthermore, if we are to advance the agenda of proof as a process through which students develop key mathematical understandings (Reid, 2011; Stylianides, Stylianides, & Weber, in press), our research lenses for mathematical proving must accommodate the specific mathematics being learned. We are neither claiming that proof is the sole process through which students learn mathematics, nor that the sole purpose of proof is for students to learn particular mathematical concepts. We are merely agreeing with prior literature that portrays proof as a process through which students learn mathematics (NCTM, 2000; Reid, 2011; Stylianides et al., in press) by pursuing the various functions of proof such as to verify, to explain, to explore, to communicate, etc. (Hanna, 2000). While we acknowledge the breadth of proof-oriented activities that students may engage in, we think it is illuminating to consider that proving mathematical claims involves, at least, (1) drawing mathematical inferences and (2) justifying such inferences. While justification is often associated with content-general mathematical relations such as citing a theorem, formal logic, finding counterexamples, etc., drawing mathematical inferences generally involves content-specific understandings. This is why we find Thompson (2013) definition of *meaning* valuable for research on proving as we shall explore in this paper: he defines the meaning of something to be the set of inferences available to a student based on thinking about that something in a particular way (see Dawkins, 2015, for examples of analysis of proving episodes attending to meanings). This merely emphasizes the rather natural point that students' mathematical inferences depend upon their understanding of the mathematical objects/properties at hand and one may not be able to account for their chain of inferences without investigating the students' meanings.

2. Motivating trends and questions

It is common to frame both the research questions and findings using these content-independent constructs such that they form informal subdomains of proof-oriented research. One can find numerous examples of studies on proof-oriented mathematical activity that make content-independent claims about

- Example use – Alcock & Ingles, 2008; Karunakaran, 2014; Sandefur, Mason, Stylianides, & Watson, 2013,
- Definition use – Alcock & Simpson, 2002; Ouvrier-Buffet, 2011,
- Proof production – Dawkins, 2012; Raman, Sandefur, Birky, Campbell, & Somers, 2009; Stylianides & Stylianides, 2009,
- Logic – Epp, 2003; Selden & Selden, 1995; and
- Understanding of proof – Sowder & Harel, 2003; Stylianou et al. , 2014.

It is not our goal to critique these studies per se, but rather to sensitize mathematics education researchers to the consequences of consistently investigating proving while downplaying the mathematical meanings that populate the arguments that students produce.

Why do many proof-oriented studies downplay mathematics content? Even if this question had one answer, no available evidence reveals it. Nevertheless, we proffer some possible explanations. One explanation is psychological. Proof's role in mathematics as a discipline and the mathematics education community's emphasis on mathematical process (e.g. NCTM, 2000) both lead researchers themselves to conceptualize proving in a specific content area such as real analysis as one instantiation of a broader phenomenon. Because we as experts see consistencies across our broad experiences with proving, we assimilate instances of proving into our general understanding.

A second explanation involves empirical findings. The growing body of evidence of students' difficulties interpreting, producing, and assessing proofs compels mathematics educators to improve proof-oriented instruction. Students perceive the transition into proof-oriented courses as a difficult transition, so it seems natural to partition such courses apart from other aspects of the curriculum (though we agree with Reid's, 2011, argument that proving should become and is becoming integrated as a ubiquitous means of mathematics teaching and learning). Because students' patterns of proving behavior that diverge from mathematical practice can be documented within multiple mathematical domains (i.e. the problems are content-general), we may falsely assume that these challenges can and should be addressed independently of particular content (the solutions are content-general).

A third explanation relates to the analytic process itself. Mathematics educators frequently use localized data to make analytic generalizations (Firestone, 1993) by constructing frameworks and in-depth characterizations of relatively few cases. While such studies rarely make explicit claims to sample-to-population generalizations (or even claims that the same student would exhibit similar patterns of proving behavior on a different proving task), it remains unclear how to situate the resulting empirical claims about student behavior. One could object that some research methodologies use case studies to make theoretical generalizations (Yin, 2014), and that the resulting theory claims applicability in wide settings. As such, research studies that test such theoretical generalizations may choose not to focus on the specific elements of a particular context. This point is not inconsistent with our argument in this paper because we do not deny the value of content-general lenses for research, but we see them as overly dominant in our field. Rather, we hope to sensitize researchers to the ways in which using such lenses implicitly justifies downplaying mathematical meaning and focuses us on particular aspects of students' proving behavior. Case studies are instances, but it is up to researchers and readers to determine what they instantiate. We, as researchers, need to be careful about explicitly resolving what each case is an instant of, and in doing so, we claim that the particulars of a given context have to considered. Furthermore, when designing and conducting studies to test existing theory, it becomes paramount that researchers examine the context of the development of the theory, and the possibly