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Performance analysis of the multi-objective ant colony optimization algorithms for the traveling salesman problem

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ABSTRACT

Most real world combinatorial optimization problems are difficult to solve with multiple objectives which have to be optimized simultaneously. Over the last few years, researches have been proposed several ant colony optimization algorithms to solve multiple objectives. The aim of this paper is to review the recently proposed multi-objective ant colony optimization (MOACO) algorithms and compare their performances on two, three and four objectives with different numbers of ants and numbers of iterations. Moreover, a detailed analysis is performed for these MOACO algorithms by applying them on several multi-objective benchmark instances of the traveling salesman problem. The results of the analysis have shown that most of the considered MOACO algorithms obtained better performances for more than two objectives and their performance depends slightly on the number of objectives, number of iterations and number of ants used.

1. Introduction

Ant Colony Optimization (ACO) was introduced by Dorigo and Stutzle in the early 1990s which is based on the behavior of natural ant colonies in particular, the foraging behavior of real ant species [1]. The indirect communication of real ants in the colony uses pheromone trail laying on the ground to find the shortest path between their food source and the nest. This procedure of real ant species in the colony is exploited by artificial ants. Moreover, ACOs are becoming popular approaches for solving combinatorial optimization (CO) problems such as the traveling salesman problem, job shop scheduling problem and quadratic assignment problem. Recently, ACO algorithms have been proposed to solve multi-objective problems (MOACO algorithms). Most of these algorithms find pareto optimal solutions and this characteristic makes ACO very attractive to solve multi-objective optimization problems.

The aim of this study is to review the recent MOACO, including pareto strength ant colony optimization (PSACO), and study the performances of those algorithms by comparing them. These MOACO algorithms have been applied to the travelling salesman problem (TSP) and six TSP problem instances have been considered to solve two, three and four objectives by changing the number of ants and number of iterations. A detailed analysis has been developed to analyze each of the MOACO algorithms by considering some of the performance indicators.

The remainder of this paper is structured as follows: some preliminaries about the multi-objective optimization problem, travelling salesman problem and ACO algorithms are reviewed in Section 2. Section 3 introduces the recent MOACO algorithms. The experimentation with the adaptation of MOACO algorithms to the traveling salesman problem, performance indicators, problem instances and parameter settings are presented in Section 4. In Section 5, the experimental results of the study are analyzed. Section 6 provides some concluding remarks.

2. Preliminaries

2.1. Multi objective optimization problem

Many real world problems consist more than one objective functions which are to be minimized or maximized simultaneously [2]. Single objective optimization problems find only one solution. However, multi-objective optimization problems find a set of optimal solutions. Generally, the multi-objective optimization can be presented as follows:

$$\left.\begin{aligned} &y = f(x) = [f_1(x), f_2(x), \ldots, f_m(x)],\\ &e(x) = [e_1(x), e_2(x), \ldots, e_k(x)] \geq 0,\\ &x = (x_1, x_2, \ldots, x_n) \in X\\ &y = (y_1, y_2, \ldots, y_m) \in Y \end{aligned}\right\} \qquad (1)$$

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where X denotes the decision space of a set of decision variables n and the objective space is denoted by Y of a set of objective functions m with k restrictions.

2.2. Traveling salesman problem

The traveling salesman problem (TSP) is an extensively studied combinatorial optimization problem by computer scientists and mathematicians. In TSP, a salesman starts from his home city and returns to the starting city by visiting each city exactly once to finding the shortest path between a given set of cities [1]. To represent the TSP, a complete weighted graph $G=(N,E)$ can be used, where a set of nodes N represents the cities and E is the set of arcs which has fully connected the nodes. A value d_{ij} is assigned for each arc $(i,j) \in E$ to represents the distance between nodes i and j. The distances between the cities are independent in the symmetric TSPs for every pair of nodes, that is, $d_{ij}=d_{ji}$. In the asymmetric TSP (ATSP), distances between the cities are not independent for at least one pair of nodes, which means, $d_{ij} \neq d_{ji}$.

2.3. Ant colony optimization algorithm

Ant colony optimization (ACO) is a meta-heuristic which has been emerged recently, for solving hard combinatorial optimization problems [1]. ACO is based on the characteristics of the real ant colonies. Ants in the colony find the shortest path for gathering food by frequently travelling between their nest and food source. When ants move between nest to the food source, they deposit a chemical called *pheromone trails* on the path which can be followed by other ants to find the shortest path to the food source. If no more pheromone is laid down the pheromone trail evaporates over time. This indirect communication behavior of real ants is based on the artificial ants.

Artificial ants find solutions by transiting from one node to another. Also, they use special data structure which stores in the memory to keep their previous actions and it is used when ants move from one node to another. Each path has constant amount of pheromone when an ant starts its journey from the first node. After the ant completes its tour by visiting the first node to the last node, the pheromone trail of all paths are updated. If the completed path by the ant is a good path, then the pheromone trail of that path will be high and vice versa. Also pheromone trails are evaporated in each path before applying the new pheromone trail. Furthermore, when the ant moves, it considers *heuristic information* which measures the quality of the given problem.

2.3.1. Ant system

Dorigo et al. [3] proposed the first ant colony optimization algorithm, Ant System (AS) for solving stochastic combinatorial optimization problems. This approach was applied to the classical traveling salesman problem and also to the asymmetric traveling salesman problem, the quadratic assignment problem and the job shop scheduling problem. All the pheromone values are associated with edges and the initial pheromone value of each edge set is equal to the given value τ_0. The heuristic information η set to $1/d_{ij}$, where d_{ij} represents the distance between the city i and j. Initially, m ants are placed into the randomly selected cities. Thereafter, every ant k moves from city i to city j using the probability given in the following equation:

$$P_{ij}^k = \begin{cases} \dfrac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{u \in N_i^k} [\tau_{iu}]^\alpha [\eta_{iu}]^\beta} & \text{if } j \in N_i^k \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The relative importance of the pheromone trail and the heuristic information are represented by the parameters α and β, respectively. N_i^k is the feasible neighborhood of ant k in city i. After n iterations all the ants have completed a tour, the pheromone trails are updated. First the pheromone trail is evaporated and then pheromones are deposited on arcs that ants have visited as in the following equation:

$$\tau_{ij} = (1-\rho)\tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^k \tag{3}$$

The pheromone evaporation rate denotes by $\rho(0 \leq \rho < 1)$ and the amount of pheromones deposited by ant k on arc (i,j) denotes by $\Delta\tau_{ij}^k$. In AS $\Delta\tau_{ij}^k$ is defined as follows:

$$\Delta\tau_{ij}^k = \frac{Q}{L_k} \tag{4}$$

where Q is a constant and L_k being the total length of the tour of the k-th ant.

2.3.2. Ant colony system

Dorigo and Gambardella [4] introduced the ant colony system (ACS) which is based on the ant system (AS) algorithm and it has improved the efficiency of AS when applied to the traveling salesman problem. Artificial ants of ACS use parallel searching procedure to find better solutions for small TSP instances. ACS identified three main modifications which differ from the Ant System.

(i) Each ant in ACS uses the state transition rule to select the next node to be visited as given in the following equation:

$$j = \begin{cases} \arg\max\limits_{j \in N_i^k} [\tau_{ij}]^\alpha [\eta_{ij}]^\beta & \text{if } q \leq q_0 \\ J & \text{otherwise} \end{cases} \tag{5}$$

where q is a random number selects in $[0,1]$ and $q \in [0,1]$ is a parameter. J is a random value calculated using the probability distribution given by Eq. (2). β is a parameter which represents the relative importance of pheromone information versus distance. When consider the Eqs. (2) and (5) together it is called pseudo-random-proportional rule.

(ii) After all ants completed their tour, the global updating rule is applied for the best ant tour from the beginning of the trail and deposit the pheromones using the following equation:

$$\tau_{ij} = (1-\rho)\tau_{ij} + \rho\Delta\tau_{ij}^{best} \tag{6}$$

The best ant deposits the pheromone $\Delta\tau_{ij}^{best} = 1/L_{gb}$, which generates the best solution. Where the length of the best ant tour from the beginning of the run denotes as L_{gb}.

(iii) After an ant moves from one node to another, the local updating rule is performed for the pheromone trail of the edge as in the following equation:

$$\tau_{ij} = (1-\rho)\tau_{ij} + \rho\Delta\tau_{ij} \tag{7}$$

where $\Delta\tau_{ij} = \tau_0$, τ_0 is the initial pheromone trail of the edge.

3. Multi objective ant colony optimization algorithms

3.1. Pareto strength ant colony optimization (PSACO)

Thantulage [5] introduced pareto strength ant colony optimization (PSACO) algorithm which is based on the first ant colony algorithm, ant system (AS). For all of the objectives it uses a same pheromone matrix while pheromone trail is updated using the domination concept as in SPEA II [6]. PSACO algorithm has been extended to solve multi-objective problems. When an ant moves from one node to another it uses the random propositional rule as defined in ant system (AS) algorithm (Eq. (2)).