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A comparative performance assessment of a set of multiobjective algorithms for constrained portfolio assets selection

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ABSTRACT

This paper addresses a realistic portfolio assets selection problem as a multiobjective optimization one, considering the budget, floor, ceiling and cardinality as constraints. A novel multiobjective optimization algorithm, namely the non-dominated sorting multiobjective particle swarm optimization (NS-MOPSO), has been proposed and employed efficiently to solve this important problem. The performance of the proposed algorithm is compared with four multiobjective evolution algorithms (MOEAs), based on non-dominated sorting, and one MOEA algorithm based on decomposition (MOEA/D). The performance results obtained from the study are also compared with those of single objective evolutionary algorithms, such as the genetic algorithm (GA), tabu search (TS), simulated annealing (SA) and particle swarm optimization (PSO). The comparisons of the performance include three error measures, four performance metrics, the Pareto front and computational time. A nonparametric statistical analysis, using the Sign test and Wilcoxon signed rank test, is also performed, to demonstrate the superiority of the NS-MOPSO algorithm. On examining the performance metrics, it is observed that the proposed NS-MOPSO approach is capable of identifying good Pareto solutions, maintaining adequate diversity. The proposed algorithm is also applied to different cardinality constraint conditions, for six different market indices, such as the Hang-Seng in Hong Kong, DAX 100 in Germany, FTSE 100 in UK, S&P 100 in USA, Nikkei 225 in Japan, and BSE-500 in India.

1. Introduction

The problem of portfolio assets selection has always been a challenging task for researchers, investors and fund managers. Markowitz set up a quantitative framework for the selection of assets in a portfolio [1,2]. In this framework, the percentage of each available asset is selected in such a way that the total profit of the portfolio is maximized, while the total risk is minimized simultaneously. The sets of portfolios of assets that yield the minimum risk for a given level of return from the efficient frontier. The optimal solution for the standard form of the Markowitz portfolio asset selection problem, which is classified as a quadratic programming model, can be obtained through exact methods, such as active set methods, interior point techniques, etc.

However, portfolio assets' selection is very complicated, as it depends on many factors such as the preferences of the decision makers, resource allocation, growth in sales, liquidity, total turnover, dividend and several other factors. Some authors have also added some practical constraints such as floor, ceiling, and cardinality to the Markowitz model that make it more realistic. The inclusion of these constraints to the portfolio assets selection problem makes it intractable even for small instances. With these constraints, the problem is a mixed integer programming with quadratic objective functions. The traditional optimization methods used to solve this problem get trapped in local minima solutions. To overcome this problem, different efficient heuristic methods are developed.

An overview of the literature on the application of meta-heuristics to the portfolio selection problem has been discussed in [3]. These methods consist of simulated annealing (SA) [4], tabu search (TS) and the genetic algorithm (GA) [5]. Tunchan [6] has applied the PSO technique to solve cardinality constrained portfolios, and the results are compared with those of the GA, TS and SA. Improved PSO algorithms have been proposed by Gao and Chu [7] for the portfolio selection problem with transaction costs. The PSO algorithm has been applied to solve the constrained portfolio selection problem, with bounds on holdings (minimum buy in threshold and maximum limit in combination), cardinality, minimum transaction lots and sector capitalization constraint [8]. Hanhong et al. [9] applied the PSO technique to solve different restricted and unrestricted risky investment portfolios, and compared it with the GA.

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The portfolio assets selection problem is intrinsically a multi-objective problem having conflicting objectives, i.e., risk and returns. But, in the above studies, the problem has been viewed as a single objective optimization problem, by considering the overall objective as a weighted sum of two objectives. Such a formulation yields multiple solutions, by suitably varying the associated weights. But the selection of the appropriate weights to get an optimal solution is a difficult task. Moreover, it requires several runs to obtain multiple solutions. To overcome these shortcomings, many researchers have applied multiobjective evolutionary algorithms (MOEAs) to solve the problem. One of the main advantages of a MOEA is that it gives a set of possible solutions in a single run, called as a Pareto optimal solution, in a reasonable amount of time [10,11]. The Pareto ant colony optimization (PACO) has been introduced for solving the portfolio selection problem [11] and its performance been compared with other heuristic approaches (i.e., Pareto simulated annealing, and the non-dominated sorting genetic algorithm) by means of computational experiments with random instances.

The portfolio assets' selection problem with many practical constraints is reported by many researchers [12–16]. Mishra et al. [12,13] have applied MOEAs to solve the portfolio assets selection problem with only budget constraint. The literature survey reveals that the cardinality constraint has been addressed by using the hybrid local search in MOEA [14]. The floor, ceiling and cardinality constraints are addressed using MOEAs by some authors [15,16]. All these aforementioned studies lack generality and in-depth analysis, in examining how the presence of these constraints affects the decision of the portfolio manager. Hence, a solution to the portfolio assets selection problem, satisfying a set of constraints, is a challenging one for researchers. In the proposed work, the combined presence of practical constraints such as the budget, floor, ceiling and cardinality is considered, to make the portfolio assets selection problem more realistic.

In the present study, the portfolio assets selection problem is formulated as a multiobjective minimization problem with four practical constraints, and is solved by using the proposed non-dominated sorting multiobjective particle swarm optimization (NS-MOPSO) algorithm. Some peer MOEAs based on non-dominated sorting, such as the Pareto envelope-based selection algorithm-II (PESA-II) [17], strength Pareto evolutionary algorithm 2 (SPEA 2) [18], non-dominated sorting genetic algorithm-II (NSGA-II) [19], and the two-*lbests* based multiobjective particle swarm optimizer (2LB-MOPSO) [20], have been applied to the problem. One MOEA algorithm based on decomposition (MOEA/D) [21] has also been applied to the same problem by formulating the portfolio asset selection problem as a multiobjective maximization problem. The performance of these MOEAs is evaluated, using four statistical metrics such as generation distance, spacing, diversity and convergence metrics. Two nonparametric statistical tests for the pairwise comparison of MOEAs are also demonstrated. The performances of these MOEAs are also compared with four those of single objective optimization algorithms such as the GA, TS, SA and PSO, using the mean Euclidean distance, variance of return error and mean return error.

The rest of the paper is organized as follows. The multiobjective optimization is presented in a concise manner in Section 2. Different multiobjective evolutionary algorithms' frameworks, and the proposed non-dominated sorting multiobjective particle swarm optimization (NS-MOPSO) are discussed in Section 3. In Section 4, the portfolio assets selection problem and its multiobjective formulation are described. Four performance metrics for assessing the performance of MOEAs are discussed in Section 5. Section 5.5 provides the experimental results of the present study. Finally, the conclusion of the investigation is presented, and further possible extension of the work is outlined in Section 6.

2. Multiobjective optimization: basic concepts and overview

Multiobjective optimization deals with the simultaneous optimization of multiple objective functions, which are conflicting in nature. A multiobjective optimization problem (MOP) is defined as the problem of computing a vector of decision variables that satisfies the constraints and optimizes a vector function, whose elements represent the objective functions. The generalized multiobjective minimization problem may be formulated [28] as

$$\text{Minimize} \quad f(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}), \ldots, f_M(\vec{x})) \tag{1}$$

Subject to constraints:

$$g_j(\vec{x}) \geq 0, \quad j = 1, 2, 3\ldots, J \tag{2}$$

$$h_k(\vec{x}) = 0, \quad k = 1, 2, 3\ldots, K \tag{3}$$

where $\vec{x}$ represents a vector of decision variables $\vec{x} = \{x_1, x_{2,} \ldots, x_N\}^T$

The search space is limited by

$$x_i^L \leq x_i \leq x_i^U, \quad i = 1, 2, 3\ldots, N \tag{4}$$

x_i^L and x_i^U represent the lower and upper acceptable values respectively for the variable x_i. N represents the number of decision variables and M represents the number of objective functions. Any solution vector $\vec{u} = \{u_1, u_2, \ldots u_K\}^T$ is said to dominate over $\vec{v} = \{v_1, v_2, \ldots, v_k\}^T$ if and only if

$$\left.\begin{array}{ll} f_i(\vec{u}) \leq f_i(\vec{v}) & \forall i \in \{1, 2, \ldots, M\} \\ f_i(\vec{u}) < f_i(\vec{v}) & \exists i \in \{1, 2, \ldots, M\} \end{array}\right\} \tag{5}$$

Those solutions which are not dominated by other solutions for a given set are considered as non-dominated solutions. The front obtained by mapping these non-dominated solutions into objective space is called the Pareto optimal front (*POF*)

$$POF = f(\vec{x}) = \{(f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})) | \vec{x} \in p\} \tag{6}$$

where p is the set of obtained non-dominated particles.

The generalized concept of the Pareto front was introduced by Pareto in 1986 [25]. The pioneering work in the practical application of the genetic algorithm to MOP is the vector evaluated genetic algorithm (VEGA) [26]. For similar applications the PESA-II [17], SPEA 2 [18], NSGA-II [19], MOEA/D [21] algorithms have been proposed by many authors. In the recent past, the heuristic approach based on particle swarm optimization has been introduced by Coello et al. [28] to solve multiobjective problems. Some other variants of multiobjective particle swarm optimization techniques, such as the TV-MOPSO [30], FCPSO [31] and 2LB-MOPSO [20] have been suggested to solve the MOP. The PSO is used in the MOEA/D framework where each particle is responsible for solving one subproblem [32]. Multiobjective evolutionary algorithms based on the summation of normalized objective values and diversified selection (SNOV-DS) for solving MOP are proposed by Qu and Suganthan [23]. Following these algorithms a variant of the multiobjective optimization algorithm using particle swarm optimization, called as non-dominated sorting particle swarm optimization (NS-MOPSO), has been proposed and employed to solve the portfolio assets selection problem.

3. Multiobjective evolutionary algorithms' frameworks

According to algorithmic frameworks, the MOEAs may be categorized as non-dominated sorting based, decomposition based, memetic, convolution-based and indicator-based [27]. In this paper, five MOEAs algorithms based on non-dominated