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Co-development of fraction magnitude knowledge and mathematics achievement from fourth through sixth grade☆

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ABSTRACT

Fraction magnitude understanding is linked to student achievement in mathematics, but the direction of the relation is not clear. To assess whether fraction magnitude knowledge and mathematics achievement develop in a bidirectional fashion, participants ($N = 536$) completed a standardized mathematics achievement test and two measures of fraction magnitude understanding—fraction comparisons and fraction number line estimation (FNLE)—twice yearly in 4th–6th grades. Cross-lagged panel models revealed significant autoregressive paths for both achievement and magnitude knowledge, indicating longitudinal stability after accounting for correlational and cross-lagged associations. Mathematics achievement consistently predicted later FNLE and fraction comparison performance. FNLE and fraction comparisons predicted mathematics achievement at all time points, although this relation diminished over time. Findings suggest that fraction magnitude knowledge and broader mathematics achievement mutually support one another. FNLE predicted subsequent mathematics achievement more strongly than did fraction comparisons, possibly because the FNLE task is a more specific measure of fraction magnitude understanding.

1. Introduction

A large body of research links student understanding of fraction magnitudes to broader mathematics achievement (Bailey, Hoard, Nugent, & Geary, 2012; Resnick et al., 2016; Siegler, Thompson, & Schneider, 2011). Fraction magnitude understanding involves the ability to comprehend, estimate, and compare the sizes of fractions (Fazio, Bailey, Thompson, & Siegler, 2014). However, the direction of the relation between mathematics achievement and fraction magnitude understanding is not clear.

Some research indicates that knowledge of fraction magnitudes provide a key underlying structure for later learning of related mathematics skills (Booth & Newton, 2012). For example, knowledge of fraction equivalence (e.g., $1/3 = 4/12$) supports learning about ratios and proportions (Siegler & Pyke, 2013). Further, understanding how the numerator and denominator of a fraction work together to determine its magnitude supports algebra learning for problems such as finding the slope of a line, measuring rates, and manipulating algebraic equations (Booth & Newton, 2012). However, it is also plausible that fraction magnitude understanding is in large part a function of prior mathematics achievement (Bailey, Siegler, & Geary, 2014; Vukovic et al., 2014). Whole number magnitude understanding, calculation fluency, multiplicative reasoning, and long division all predict later development of fraction knowledge, including fraction magnitude understanding (Hansen et al., 2015; Jordan et al., 2013; Vukovic et al., 2014).

The possibility of a bidirectional relation between fraction magnitude understanding and broad mathematics achievement—that is, a relationship in which learning in one area supports learning of the other, and vice versa—has not been fully investigated. Modest indirect evidence of such a bidirectional relation comes from Watts et al. (2015), who examined the effect of first-grade mathematics achievement on later mathematics achievement at age 15. Results showed that fraction knowledge in fifth grade mediated this relationship, suggesting that early mathematics achievement facilitates fraction learning, which in turn bolsters mathematics achievement more broadly. However, data in the form of repeated, concurrent measures of fraction knowledge and mathematics achievement is necessary to more rigorously test for a longitudinal, mutually supportive relationship.

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1.1. Cross-lagged panel models of relationships between numerical knowledge and mathematics achievement

Cross-lagged panel models have been used to examine the directionality of the relation between whole number knowledge and broad mathematics achievement from kindergarten to second grade (Friso-van den Bos et al., 2015), as well as the relation between fraction knowledge and broad mathematics achievement in sixth and seventh grade (Bailey et al., 2012). Cross-lagged panel modeling provides better evidence of directionality than other correlational and mediation designs, as this approach allows for the evaluation of cross-domain lagged effects while controlling for effects of prior achievement within each domain (i.e., autoregressive effects). Thus, cross-lagged models can be used to analyze phenomena longitudinally and investigate potential causality in both directions (Bollen & Curran, 2006).

Friso-van den Bos et al. (2015) used cross-lagged panel analyses to examine the relation between whole number magnitude understanding, measured by a whole number line estimation (WNLE) task (using a 0–100 number line) and mathematics achievement (measured by a standardized achievement test covering a wide range of mathematics topics) over four time points in first and second grades. It was shown that WNLE ability and general mathematics achievement develop in tandem over the course of first and second grades. For example, as children solve general arithmetic problems, they may use a mental number line (e.g., Halberda, Mazzocco, & Feigenson, 2008; Siegler & Booth, 2004), reasoning, for example, that 21 minus 13 is unlikely to be 34, because 34 is to the right of 21 on the number line. This, in turn, improves their understanding of numerical magnitudes (Friso-van den Bos et al., 2015).

Bailey et al. (2012) employed a cross-lagged panel model to investigate the relation between fraction magnitude knowledge (measured by a comparison task that prompted students to choose the larger of two fractions) and broad mathematics achievement in sixth and seventh grades. Fraction magnitude knowledge predicted one-year gains in mathematics achievement, but the opposite predictive relationship was not significant. In other words, prior fraction magnitude knowledge appeared to underlie gains in mathematics achievement between sixth and seventh grade, but prior mathematics achievement did not influence development of fraction magnitude skills during the same time period. Bailey and colleagues speculate that the unidirectional relationship reflects a causal mechanism whereby the development of fraction magnitude knowledge may help students learn decimals, ratios and proportions, and pre-algebraic reasoning, all key topics that form the foundation for learning algebra. However, the generalizability and validity of the findings are limited by the study's assessment of only two time points, both of which occurred *after* the bulk of fractions instruction took place in fourth through sixth grade (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

We hypothesize that, had Bailey et al. (2012) assessed directional relationships earlier in development, evidence of a bi-directional relationship might have emerged. Previous research (e.g., Vukovic et al., 2014) indicates that a myriad of whole number skills predicts later fraction knowledge (including fraction magnitude understanding). Additionally, research in the earlier grades (e.g., Friso-van den Bos et al., 2015) indicates that there is a bi-directional relation between whole number magnitude knowledge and mathematics achievement. As such, magnitude knowledge seems to develop iteratively with mathematics achievement in elementary school, in contrast to what was found by Bailey et al. (2012) with older students. One issue may be that Bailey et al. used only a brief fraction comparison task to assess fraction magnitude knowledge; this task may reflect knowledge of procedures instead of or in addition to magnitude understanding. Thus, it is possible a bi-directional relationship might be observed for a different measure of magnitude knowledge, such as fraction number line estimation (FNLE).

Previous research on the relation between fraction magnitude understanding and mathematics achievement has been limited both in quantity and scope; studies have focused on older students (e.g., Bailey et al., 2014), relied on cross-sectional designs (e.g., Siegler & Pyke, 2013), or examined whether fraction understanding predicts much later mathematics outcomes in high school (e.g., Siegler et al., 2012). To date, no longitudinal research has used multiple measures to thoroughly examine the potential for a bi-directional relation between fraction magnitude knowledge and mathematics achievement during the important period when primary instruction in fractions takes place in school.

2. The present study

Our study aims to test our hypothesis that the reciprocal relationship witnessed between whole number magnitude understanding and mathematics achievement in early schooling (Friso-van den Bos et al., 2015) also holds for fraction magnitude knowledge and mathematics achievement in the intermediate grades. First, we hypothesize that, as was found by Bailey et al. (2012), fraction magnitude understanding will predict broader math achievement. Fraction knowledge is likely to support acquisition of other more complex mathematics topics in fifth grade, such as knowledge of long division and decimals. However, in contrast to the findings of Bailey et al., we also expect that general mathematics ability (e.g., whole number knowledge) will support fraction magnitude understanding, as proficiency in other areas of mathematics seems likely to help students learn fractions. For example, in fourth grade, students learn about multiplicative relationships (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), which would then help them reason about relationships among fraction magnitudes (e.g., one-half is the same as two-fourths, and both numbers are represented in the same location on the number line). The notion of a bidirectional relationship between fraction magnitude knowledge and general mathematics achievement is consistent with Siegler's integrated theory of numerical development (Siegler et al., 2011; Siegler & Lortie-Forgues, 2014), which holds that numerical learning is a progressive broadening of the set of numbers whose magnitudes can be accurately represented. Thus, finding a mutually predictive relationship would provide further evidence for the continuous role of numerical magnitude knowledge in learning increasingly complex mathematics, such as algebra (e.g., reasoning about linear equations (Booth, Newton, & Twiss-Garrity, 2014)).

We assessed students on fraction magnitudes and mathematics achievement at two time-points per grade, resulting in a total of six measurements. Given that no prior research has focused on the relationship between fraction magnitude understanding and mathematics achievement between fourth and sixth grade, despite this being the period during which most fraction instruction takes place, the present study fills an important gap in the literature. Our study also differs from previous work by controlling for a broader range of general cognitive abilities that may influence fraction knowledge and general mathematics achievement. Previous research has either not controlled for cognitive factors at all (Friso-van den Bos et al., 2015) or has included only a few variables (Bailey et al., 2012), even though recent analyses suggest that cognitive factors (e.g., intelligence and working memory) significantly predict the development of children's broad mathematics achievement over time (Bailey, Watts, Littlefield, & Geary, 2014). Failure to control for these variables could potentially confound results regarding the relationship between fractions ability and math achievement. Selection of the control variables used in the present study was guided by prior research on predictors and correlates of mathematical achievement (Geary, 2004; Gunderson, Ramirez, Beilock, & Levine, 2012; Swanson, 2011) and fraction knowledge in particular (Hecht, Close, & Santisi, 2003; Hecht & Vagi, 2010; Jordan et al., 2013; Seethaler, Fuchs, Star, & Bryant, 2011). We control for background variables, including age, gender, and income status, as well