



The use of number-based versus digit-based strategies on multi-digit subtraction: 9–12-year-olds' strategy use profiles and task performance



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ABSTRACT

We examined children's number-based and digit-based strategy use profiles on multi-digit subtraction problems in relation to task performance and individual (gender, grade, mathematical achievement level) and contextual (country) characteristics. 318 third- to sixth-graders solved a multi-digit subtraction strategy use task, where half of the subtraction problems had number characteristics that stimulated the number-based compensation strategy. First, latent class analyses revealed five strategy use profiles. Most children consistently relied on one strategy (either digit-based strategies or the number-based decomposition or sequential strategies) across problems. Only a minority of children demonstrated strategy variety and flexibility – more often high achievers and children from higher grades. Second, children's strategy instruction history was related to their profile of strategy use, but this relation was less straightforward than generally assumed. Finally, consistent digit-based strategy users were more accurate than consistent decomposition strategy users and varied number-based strategy users.

1. Introduction

Children's thinking is characterized by variability, at all ages and in different cognitive domains (Siegler, 2000, 2003, 2007). This variability in thinking, expressed in variability in the range of strategies that are used to solve cognitive tasks, makes it necessary to flexibly choose between different strategies. Strategy variability and strategy flexibility are important stepping stones towards improved task performance and later learning, also in the domain of mathematics (Siegler, 2000, 2003, 2007). Moreover, according to the adherents of the latest international reform movement in mathematics education, strategy variability and strategy flexibility are important components of mathematical competence (Baroody & Dowker, 2003; Kilpatrick, Swafford, & Findell, 2001; Verschaffel, Greer, & De Corte, 2007). Concretely, in their view, mathematical competence is much more than being able to answer mathematics tasks accurately. Acquiring competence in using a rich diversity of efficient strategies and being able to apply these strategies in real-world problem solving are considered as equally important components of developing mathematical expertise (e.g., McMullen, Brezovszky, Hannula-Sormunen, & Lehtinen, 2016).

Against this background, an ever-growing number of researchers all over the world have investigated children's strategy variety and strategy flexibility in the domain of single-digit arithmetic (for reviews, see Kilpatrick et al., 2001; Verschaffel et al., 2007). These studies

converged on the conclusion that with increasing experience in the domain, children use a rich variety of strategies to solve elementary arithmetic tasks, and flexibly apply these strategies on different types of problems. In contrast, children's strategy variety and flexibility in the domain of multi-digit arithmetic has attracted only limited attention in research (for reviews, see Kilpatrick et al., 2001; Verschaffel et al., 2007). The absence of studies about multi-digit arithmetic is problematic. First, because it is an important domain in the upper grades of elementary school, and, second, because the range of strategies that can be flexibly applied becomes more varied with the introduction of the digit-based algorithm.

The current study aimed to scrutinize and systematically analyse children's solution strategies in solving multi-digit subtractions, i.e., three-digit subtractions with numbers up to 1000. To adequately capture the variability both between and within children, we used an advanced individual-differences perspective. With latent class analysis we aimed to characterize children's strategy variability and flexibility by identifying different profiles of strategy use across a varied set of subtraction problems. To maximize the occurrence of variability, we included children of varying grades (third- to sixth-graders), mathematical achievement levels (low to high achievers) and type of strategy instruction. In view of the ecological validity of the present study, we operationalized the type of strategy instruction via the country children were educated in, namely Flanders (Belgium) and the Netherlands. As

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outlined below, these countries are known to be highly similar in terms of their general cultural and educational background characteristics, but clearly differ with respect to multi-digit arithmetic strategy instruction.

1.1. Multi-digit subtraction: Types of strategies

Children apply two major types of strategies to solve multi-digit subtractions in the number domain up to 1000, namely number-based strategies and digit-based algorithms (see Fuson, 2003; Kilpatrick et al., 2001; Verschaffel et al., 2007, for reviews). Number-based strategies can be defined as calculation methods operating on the numerical values of the integers in the problem and relying on one's understanding of the basic features of the number system and of arithmetic operations, a well-developed feeling for numbers and a sound knowledge of the elementary number facts (Anghileri, 1999; Buys, 2001; Thompson, 1999a). Although these strategies can be executed in children's heads, without using paper and pencil, children may also write down their calculation steps and/or intermediate results during the solution process.

Children's number-based strategies for multi-digit subtraction can be further classified into three basic categories (Buys, 2001; Verschaffel et al., 2007). The first category involves *decomposition* strategies, that are characterized by first decomposing the minuend and the subtrahend into their decimal structure, as when splitting off the hundreds (H), tens (T) and units (U) in both the minuend and the subtrahend, and next subtracting them separately (e.g., $457 - 298 = _$; $400 - 200 = 200$, $50 - 90 = -40$, $7 - 8 = -1$, so the answer is $200 + (-40) + (-1) = 159$). The second category includes *sequential* strategies, defined as decomposing the subtrahend decimally and subtracting these parts from the un-split minuend, as in taking away first the subtrahend's H, next its T and finally its U from the un-split minuend (e.g., $457 - 298 = _$; $457 - 200 = 257$, $257 - 90 = 167$, $167 - 8 = 159$, so the answer is 159). The third category, *varying* strategies, refers to diverse clever strategies that involve the flexible adaptation of the numbers and/or operations in the problem on the basis of one's understanding of number relations and/or the properties of arithmetic operations. An example of the latter type of strategies is the compensation strategy, which is considered to be particularly efficient for subtractions with a subtrahend with a U-value 8 or 9 (e.g., $457 - 298 = _$ via $457 - (300 - 2) = 157 + 2 = 159$).

Besides number-based strategies, children also learn to solve multi-digit subtractions by means of digit-based standard algorithms. These algorithms are fixed and well-defined step-by-step procedures, involving operations with the digits of the numbers in the problem (rather than the numerical value of these digits), such as calculating the difference between 5 and 9 (rather than 50 and 90) and between 4 and 2 (instead of 400 and 200) when solving $457 - 298 = _$. Although digit-based standard algorithms are usually executed on paper, it is, at least in principle, also possible to apply them in one's head, without any reliance on writing materials.

1.2. Number-based versus digit-based strategies: Previous studies in the domain of multi-digit subtraction

Although the latest international reform movement in mathematics education focuses on strategy variety and strategy flexibility, children's use of number-based versus digit-based strategies for solving multi-digit subtractions received only limited research attention (for reviews, see Kilpatrick et al., 2001; Verschaffel et al., 2007). Consequently, our insights into children's strategy competencies in this domain are limited.

The available research revealed little variety and consequently also limited flexibility in children's strategy choices in the domain of multi-digit subtraction (Heinze, Marschick, & Lipowsky, 2009; Selter, 2001; Torbeyns & Verschaffel, 2013, 2015). Before the introduction of the digit-based strategy, children tend to consistently rely on one type of

number-based strategies, particularly decomposition or sequential strategies to solve subtractions up to 1000 (Heinze et al., 2009; Selter, 2001). After the introduction of the digit-based strategy, children are found to heavily rely on this digit-based algorithm (Selter, 2001; Torbeyns & Verschaffel, 2013, 2015). As a result, there is hardly any variety and flexibility in children's strategy use, as they tend to apply either only number-based decomposition or sequential strategies (before the introduction of the digit-based algorithm) or the digit-based strategy (after the introduction of the digit-based algorithm) on all types of subtraction problems, ignoring the specific numerical features of the problem (Selter, 2001; Torbeyns & Verschaffel, 2013, 2015).

Turning to the efficiency of number-based versus digit-based strategies on subtraction problems in the number domain up to 1000, the available findings are mixed (Selter, 2001; Torbeyns & Verschaffel, 2013, 2015). The results of Selter (2001) seriously question the efficiency of the digit-based strategy for solving multi-digit subtractions, which is in line with the findings of older studies on children's use of so-called 'buggy procedures' (incorrect variations of the standard algorithm; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Fuson et al., 1997; Thompson, 1999b, 1999c). In contrast, Torbeyns and Verschaffel (2013, 2015) provided evidence for the superior efficiency of the digit-based strategy compared to number-based strategies, with more accurate and faster task performance for the former than the latter type of strategies.

Although these few available studies already provide some first insights into children's strategy competencies in the domain of multi-digit subtraction, they are constrained by three weaknesses. A first weakness relates to the conceptualization of children's number-based and digit-based strategy use as a purely scalar, numerical variable, ignoring the *qualitative differences* in children's strategy use (Hickendorff, van Putten, Verhelst, & Heiser, 2010; Yang, Shaftel, Glasnapp, & Poggio, 2005). Starting from a person-centred view on the acquisition of mathematical knowledge and skill, Hickendorff and colleagues recently started to explore patterns of qualitative differences in children's strategy competencies in the domain of multi-digit arithmetic using latent class analysis (Fagginger Auer, Hickendorff, & van Putten, 2016; Fagginger Auer, Hickendorff, van Putten, Béguin, & Heiser, 2016; Hickendorff, Heiser, van Putten, & Verhelst, 2009; Hickendorff et al., 2010). These studies revealed that individual differences in strategy use can better be characterized by qualitatively different strategy use profiles capturing variability between and within children, instead of as quantitative variables (i.e., counting the number of times a strategy is used on a set of problems and subjecting that number to regression-like techniques). For instance, these studies showed that there are children who consistently rely on one (type of) strategy and children who flexibly switch among different (types of) strategies across problems.

A second weakness of the previous investigations is that children's variety and flexibility in number-based and digit-based strategy use were not analysed *in direct relation to their performance* on the multi-digit subtraction task. As discussed in Lemaire and Siegler's (1995) model of strategy change, children's task performance is directly associated with the types of strategies they apply (strategy repertoire), the frequency of using the different types of strategies (strategy distribution), the accuracy and speed of strategy execution (strategy efficiency) and the flexibility of strategy choices (strategy selection). Improvements in each of these four strategy parameters are assumed to result in increases in children's task performance. However, the available efficiency results ignore children's profile of strategy use across the total problem set. Therefore, these studies do not provide insights into the relation between children's strategy variety and flexibility and their overall performance in the domain of multi-digit subtraction.

As a third weakness of the previous studies, we refer to the scarce attention for the contribution of individual characteristics to strategy development, including children's gender, grade and mathematical achievement level, and of contextual variables, including children's strategy instruction. Although investigations in the domain of multi-

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