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The role of domain-general cognitive abilities and decimal labels in at-risk fourth-grade students' decimal magnitude understanding☆

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ABSTRACT

The purpose of the study was to determine whether individual differences in at-risk 4th graders' language comprehension, nonverbal reasoning, concept formation, working memory, and use of decimal labels (i.e., place value, point, incorrect place value, incorrect fraction, or whole number) are related to their decimal magnitude understanding. Students (n = 127) completed 6 cognitive assessments, a decimal labeling assessment, and 3 measures of decimal magnitude understanding (i.e., comparing decimals to the fraction $\frac{1}{2}$ benchmark task, estimating where decimals belong on a 0–1 number line, and identifying fraction and decimal equivalencies). Each of the domain-general cognitive abilities predicted students' decimal magnitude understanding. Using place value labels was positively correlated with students' decimal magnitude understanding, whereas using whole-number labels was negatively correlated with students' decimal magnitude understanding. Language comprehension, nonverbal reasoning, and concept formation were positively correlated with students' use of place value labels. By contrast, language comprehension and nonverbal reasoning were negatively correlated with students' use of whole number labels. Implications for the development of decimal magnitude understanding and design of effective instruction for at-risk students are discussed.

1. Introduction

Many students struggle when the curriculum shifts from whole numbers to rational numbers in the upper elementary grades. The ability to accurately assess magnitude is thought to be key for consolidating properties of whole numbers and rational numbers, since magnitude is a unifying property of *all* numbers (Siegler, Thompson, & Schneider, 2011) and rational number magnitude knowledge is related to future mathematics achievement (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Siegler, 2008; DeWolf, Bassock, & Holyoak, 2015; Fazio, Bailey, Thompson, & Siegler, 2014; Siegler et al., 2012; Siegler & Pyke, 2013). Students at risk for mathematics difficulties demonstrate pervasive and systematic misconceptions related to estimating rational number magnitude (e.g., Jordan, Resnick, Rodrigues, Hansen, & Dyson, 2016; Malone & Fuchs, 2017), but much of the research has centered on common fractions $\left(\text{i. e.}, \frac{a}{b}\right)$. It is unclear whether the development of decimal magnitude understanding among at-risk students, the focus of the present study, parallels that of fraction magnitude understanding. Understanding individual differences in at-risk students' development of decimal magnitude understanding provides insight into the cognitive abilities required to develop competence with decimals, which in turn can guide the design of early screening tools and interventions.

Most college and career-ready state standards emphasize decimal magnitude understanding. By end of fourth grade, students should be able to compare decimal tenths and hundredths and reason about their size. However, 67% of fourth-grade students could not estimate the location of a decimal on a number line on the National Assessment of Education Progress (U.S. Department of Education, 2011). Many students incorrectly apply whole-number logic to decimals, e.g., assuming 0.274 is > 0.83 because 274 is > 83 (Rittle-Johnson, Siegler, & Alibali, 2001), and these misconceptions are difficult to correct (Kallai & Tzelgov, 2014; Resnick et al., 1989; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2004).

In this paper, a *decimal* refers to a number written with digits to the right of the decimal point (e.g., 0.25). Decimal magnitude understanding refers to the ability to estimate and reason about the size of a decimal. For this study, we focused on determining the location of a decimal on a number line, rationalizing about the size of a decimal compared to the benchmark fraction $\frac{1}{2}$, and judging the validity of decimal and fraction

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equivalencies. The purpose of the study was to determine whether individual differences in cognitive abilities and use of decimal labels (i.e., place value, point, incorrect place value, incorrect fraction, or whole number) are related to at-risk fourth-graders' decimal magnitude understanding.

1.1. Potential cognitive predictors of decimal magnitude understanding

We focus on individual differences in language comprehension, nonverbal reasoning, concept formation, and working memory, as these four cognitive resources are related to the development of fraction understanding (e.g., Hecht & Vagi, 2010; Jordan et al., 2013; Namkung & Fuchs, 2015; Seethaler, Fuchs, & Star, 2011; Vukovic et al., 2014), and we located no prior studies examining cognitive predictors of decimal magnitude understanding. Although fractions and decimals have different symbolic notation (e.g., $\frac{1}{2}$ vs. 0.5) and labeling convention (e.g., "one-half" vs. "five-tenths"), they also have similar properties (i.e., both are rational numbers that can signify magnitudes less than one) and students tend to struggle with both fractions and decimals (e.g., Kallai & Tzelgov, 2009, 2014; Ni & Zhou, 2005).

In the present study, we operationalized language comprehension as the ability to accurately define printed words or use a word to describe a picture. To index nonverbal reasoning, we assessed the ability to solve logical puzzles and define relationships between pictures. For concept formation, we focused on the ability to apply a rule to a pattern of objects. For working memory, we focused on span tasks assessing the central executive component of working memory, or the ability to hold pieces of information in the mind while performing cognitive tasks. We focus on span tasks involving both sentences and numbers, as both forms have been found to be related to fraction understanding and mathematics achievement in general. The cognitive abilities incorporated in the studies described below are consistent with these methods for operationalizing these four cognitive processes.

Seethaler et al. (2011) found that language comprehension, nonverbal reasoning, concept formation, and working memory were unique predictors of fraction-calculation skill. Although the present study is not about calculations, research suggests that improved magnitude understanding improves calculation skill (e.g., Fuchs et al., 2014), as these processes likely develop iteratively (Rittle-Johnson & Siegler, 1998; Rittle-Johnson et al., 2001). Namkung and Fuchs's (2015) findings support this. They found that language comprehension, concept formation, and nonverbal reasoning play a role in fourth-grade students' development of accurate fraction number line estimation. Similarly, in a two-year longitudinal study, Jordan et al. (2013) found that third-grade students' language comprehension and nonverbal reasoning, (along with calculation fluency, reading fluency, and attentive behavior) predicted their development of conceptual understanding of fractions in fourth grade, including the ability to estimate fraction magnitude on the number line.

Vukovic et al. (2014) found a somewhat more nuanced set of relations among these domain-general abilities and the development of fraction magnitude understanding. First-grade students' language comprehension, nonverbal reasoning, working memory, and attentive behavior, were measured along with their whole-number knowledge. In second grade, students' whole-number knowledge was again indexed; then students' understanding of fractions was examined in fourth grade. Language comprehension, working memory, and attentive behavior predicted fraction understanding, including the ability to estimate fractions on the number line. Yet, although these domain-general abilities predicted fourth graders' understanding of fractions, these effects were completely mediated by students' second-grade whole-number skill.

Although their findings stand in contrast to the earlier studies, language comprehension, working memory, and attentive behavior have also been found to predict whole-number calculation skill (e.g., Seethaler & Fuchs, 2006; Seethaler et al., 2011). In the Vukovic et al. (2014) study, these domain-general abilities did not predict rational number knowledge beyond whole-number calculation skill (which is why we control for students' whole-number knowledge in the present study). It stands to reason that these abilities are important for developing competence with both whole numbers and rational numbers.

Despite some inconsistency in findings across these studies, language comprehension, nonverbal reasoning, concept formation, and working memory appear related to the development of fraction understanding, especially developing number line estimation skill. We therefore hypothesized a similar developmental pattern for decimals as fractions. At the same time, important distinctions may emerge, considering that decimals and fractions have different symbolic notation, which affects labeling conventions. For labeling fractions, the numerator and denominator have a special term (e.g., read $\frac{1}{2}$ as "one-half and $\frac{1}{3}$ as "one-third"). By contrast, labeling decimals reflects place value (e.g., read 0.2 as "2 tenths" and 0.35 as "35-hundredths"). Therefore, the bipartite $\left(\frac{a}{b}\right)$ structure of fractions may impose additional cognitive demands for estimating magnitude over what is involved for decimals (e.g., DeWolf, Grounds, Bassok, & Holyoak, 2014). Although our study did not compare and contrast fraction and decimal labeling conventions, we did investigate whether decimal labels are related to students' decimal magnitude understanding and cognitive abilities, which indirectly addresses these differences.

1.2. Labeling decimals

We identified one study that suggests using decimal place value labels is related to increased magnitude understanding. Mazzocco and Delvin (2008) investigated whether low-achieving, typically-achieving, and learning disabled sixth graders' decimal magnitude understanding and knowledge of decimal place value labels (e.g., reading 0.49 as "forty-nine hundredths") predicted their ability to rank order fractions and decimals at eighth grade. Students with mathematics learning disabilities had the most difficulty labeling decimals with place value labels, and these difficulties persisted into eighth grade. By contrast, incorrectly labeling decimals in sixth grade for low-achieving and typically-achieving students were not predictive of students' ability to name decimals with place value labels in eighth grade. That is, some low-achieving and typically-achieving students failed the naming test in sixth grade, but passed in eighth grade. This is likely because 94% of students who mastered the ranking test in eighth grade used some place value labels for decimals in sixth grade. They concluded that the inability to correctly name a decimal with place value labels may represent a key deficit in rational number understanding among students with mathematics learning disabilities, which suggests that decimal labels may play an important role in students' development of rational number understanding. The authors did note, however, that using place value labels in sixth grade did not guarantee greater magnitude understanding in eighth grade, as 22% of students who used place value labels in sixth grade failed the ranking test in eighth grade.

There is also some evidence in the fraction literature to suggest a positive relation between the quality of fraction labels and developing fraction knowledge. Miura, Okamoto, Vlahovic-Stetic, Kim, and Han (1999) compared U.S., Croatian, and Korean first- and second-graders' initial fraction ideas. They hypothesized that Korean students would have the greatest foundational knowledge of fractions because the Korean naming system for fractions directly supports magnitude understanding. That is, the direct translation of a unit fraction such as $\frac{1}{3}$ is "of three parts, one," whereas English and Croatian refer to it as "one-third," which does not produce a direct mental image of magnitude. As expected, Korean students had greater foundational knowledge of fractions than students in the other two countries. So, explicit vocabu-