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# Cognitive and linguistic predictors of mathematical word problems with and without irrelevant information☆

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ABSTRACT

The purpose of this study was to identify cognitive and linguistic predictors of word problems with versus without irrelevant information. The sample was 701 2nd-grade students who received no specialized intervention on word problems. In the fall, they were assessed on initial arithmetic and word-problem skill as well as language ability, working memory capacity, and processing speed; in the spring, they were tested on a word-problem measure that included items with versus without irrelevant information. Significant predictors common to both forms of word problems were initial arithmetic and word problem-solving skill as well as language and working memory. Nonverbal reasoning predicted word problems with irrelevant information, but not word problems without irrelevant information. Findings are discussed in terms of implications for intervention and future research.



## 1. Introduction

Word problems (WPs) represent a major component of the mathematics curriculum across kindergarten through high school, and many high-stakes standardized tests, such as the National Assessment of Educational Progress (NAEP; National Assessment Governing Board, 2009), place heavy emphasis on mathematical word-problem solving. This makes sense because among school-age math measures, WPs are the best predictor of adult employment and wages (Every Child a Chance Trust, 2009; Murnane, Willett, Braatz, & Duhaldeborde, 2001; Parsons & Bynner, 1997; Rivera-Batiz, 1992). Therefore, improving word-problem solving is critical for school and occupational success.

Not surprisingly, students at risk for or with mathematics learning disabilities struggle with word-problem solving (Parmar, Cawley, & Frazita, 1996). More surprising is that this struggle often occurs in the presence of adequate arithmetic skill (Fuchs et al., 2008; Swanson, Jerman, & Zheng, 2008). Some research indicates that arithmetic and word-problem solving are distinct components of mathematical competence. For example, studies demonstrate that the cognitive and linguistic processes underlying word-problem solving differ from those involved in arithmetic (e.g., Swanson & Beebe-Frankenberger, 2004; Fuchs et al., 2005; Fuchs et al., 2006; Geary, Hoard, Nugent, & Bailey, 2012).

For these reasons, early screening and intervention procedures for preventing WP difficultly likely require a different approach than is needed for arithmetic. Toward this end, understanding the WP features that create challenge is critical. In the present study, we focused on one potentially critical WP feature: the presence of irrelevant information. Specifically, we examined whether the cognitive and linguistic student characteristics that predict WP solution accuracy differ for WPs with versus without irrelevant information. We focused on second grade because individual differences in word-problem solving are established at this time (Fuchs et al., 2013) and because second grade is often when identification of learning disability begins (Fletcher, Lyon, Fuchs, & Barnes, 2006).

In this introduction, we begin by providing background information on word-problem solving, including a brief discussion about why word-problem solving may represent challenge in the presence of adequate arithmetic skill and about which cognitive and linguistic factors distinguish between word-problem solving and arithmetic skill. We then turn our attention to complex WPs, specifically those with irrelevant information, and provide a rationale for the present study's focus.

### *1.1. Word-problem solving: A distinct area of mathematical competence*

A major distinction between word-problem solving and arithmetic is the addition of linguistic information, which requires students to decipher a WP narrative in order to build a problem model, identify the missing information, construct a number sentence to find the missing information, and (only then) perform calculation procedures to find

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the missing number. By contrast, arithmetic problems are already set up for calculation.

Four large-scale studies have examined whether arithmetic and word-problem solving skills constitute a single ability or are distinct areas of mathematical competence, by examining whether the cognitive and linguistic factors underlying word-problem solving and arithmetic differ. In these studies, simple WPs were defined as linguistically presented one-step problems that required arithmetic solutions. Studying 353 first, second, and third graders, Swanson and Beebe-Frankenberger (2004) found that short-term memory and fluid intelligence were unique to simple WPs, whereas phonological processing was unique to arithmetic. Working memory contributed strongly to both areas. Fuchs et al. (2005) measured cognitive abilities at the beginning of first grade to predict the development of arithmetic and simple WP skill among 272 children. Common predictors were teacher ratings of attentive behavior and working memory. Nonverbal problem solving was unique to simple WPs and phonological processing was unique to arithmetic. With 312 third graders, Fuchs et al. (2006) examined the cognitive correlates of arithmetic versus simple WPs while controlling for the role of arithmetic skill in simple WPs. For simple WPs, nonverbal problem solving, sight word efficiency, language, and concept formation were unique, whereas for arithmetic, processing speed and phonological decoding were unique. Only teacher ratings of attentive behavior were common to both word-problem solving and arithmetic.

With a representative sample of 924 third graders classified as having difficulty with arithmetic, word-problem solving, both domains, or neither, Fuchs et al. (2008) explored patterns of difficulty in arithmetic and word-problem solving. Students were assessed on three measures of word-problem solving and three measures of arithmetic skill, as well as nine cognitive/linguistic dimensions. Using multivariate profile analyses, Fuchs et al. found that specific arithmetic difficulty was associated with deficits in processing speed and attentive behavior and strengths in language. By contrast, word-problem solving was associated with deficits in language. Across these studies, results suggest that individual differences in word-problem solving are associated with a distinctive set of cognitive and linguistic abilities.

### *1.2. Sources of WP difficulty*

As mentioned, the most transparent distinction between word-problem solving and arithmetic is inclusion of linguistic information. The presentation of linguistic information, or the manner in which WPs are worded, influences the difficulty of a problem (e.g., Helwig, Rosek-Toedesco, Tindal, Heath, & Almond, 1999). It is important, however, to consider the type and extent of linguistic complexity embedded in a WP and to identify the sources of difficulty that make WPs especially challenging. These sources of WP difficulty may not necessarily distinguish word-problem solving and arithmetic, but instead create differential challenge within word-problem solving, as a function of WP features.

WP features that increase complexity include the following. First, the need to analyze other data sources, such as graphs or signage, to find the relevant information required for problem solving can increase challenge (Fuchs & Fuchs, 2007), by taxing working memory capacity. Second, the inclusion of irrelevant information decreases WP accuracy and causes differential challenge for students with mathematics learning disabilities (Parmar et al., 1996). In this paper, we refer to WPs that have one or more of these features as *complex WPs*. These features create problems that more accurately reflect real-world word-problem solving situations than simple WPs (i.e., WPs without these complicating features). To date, no studies have examined the cognitive predictors of complex versus simple WPs.

Complex WPs, such as those containing irrelevant information, often make problems for which solution strategies have been learned appear novel and confusing. In the present study, we focus on three problem types: (a) combine WPs (two quantities are combined to form a total), (b) compare WPs (two quantities are compared to find a difference), and (c) change WPs (an action triggers an increase or decrease in a starting amount). Now consider this simple combine WP: *Emma has two cats. Molly has three dogs. How many animals do the girls have altogether*? Next, consider this complex combine WP, with the same cover story except for the addition of irrelevant information: *Emma has two cats. Molly has three dogs. Molly walks her dogs four times a week. How many animals do the girls have altogether*? Although this irrelevant information (i.e., *Molly walks her dogs four times a week*) does not alter the problem type or the required solution method, it does make it harder for the student to identify the problem as belonging to the combine problem type. This is because most students expect combine problems to incorporate two given numbers along with one missing number. To make this situation more problematic, many students approach WPs without thinking deeply about how irrelevant information detracts them from recognizing a novel problem as belonging to a known, or previously taught, problem type. Furthermore, students may encounter irrelevant information presented within tables and graphs. For example, on many high-stakes assessments, irrelevant information is not frequently presented within problem text. In many cases, students have to negotiate irrelevant information provided within tables and graphs.

Because school instruction does little to vary the complexity of WPs, students often have difficulty deciphering problems that incorporate the kinds of complexity reflected in the real world. In other words, much instruction fails to link complex WPs to simple WPs by providing students with explicit strategies to connect complex problems with the problems used for instruction. One approach for promoting word-problem solving in school-age children that targets complex WPs is schema-broadening instruction (e.g., Fuchs et al., 2003a). Students are taught to transfer their knowledge of problem types to recognize complex problems as belonging to a problem type for which they have learned a solution strategy.

Cooper and Sweller (1987) identified three variables contributing to word-problem solving transfer. Students must (a) master problem solution rules, (b) develop categories, or *schemas*, for classifying problems that require similar solution methods, and (c) connect novel (or complex) problems to previously solved problems. Schemas facilitate transfer because students are able to connect novel problems with taught problems and apply learned problem-solution methods. In our previous example (i.e., *Emma has two cats. Molly has three dogs. How many animals do the girls have altogether*?), students learn to categorize this WP as a *combine* problem type, or schema, and then to apply a set combine solution strategies. Next, complex WP statements are introduced (i.e., *Emma has two cats. Molly has three dogs. Molly walks her dogs four times a week. How many animals do the girls have altogether*?), and intervention focuses on strategies for recognizing complex WP features, such as irrelevant information or combinations of problem types or finding relevant information in sources other than the WP statement. Students learn to connect this problem with the previous WP. They learn that while some sources of complexity make a problem appear different from what is expected, the underlying structure (i.e., problem-type) and problem solution method remain the same. The broader the schema, the greater the probability the student will recognize the connection between complex and previously solved problems.

Salomon and Perkins (1989) provided a framework for understanding how to broaden schemas and distinguished between two forms of transfer. Low-road transfer, accomplished through varied and extensive practice, involves the automatic triggering of well-learned, stimulus-controlled behavior in a new context (Fuchs et al., 2003a; Salomon & Perkins, 1989). In contrast, word-problem solving represents high-road transfer, which requires individuals to abstract connections (i.e., word-problem solving schemas) between familiar and novel tasks (Fuchs et al., 2003a; Salomon & Perkins, 1989). Salomon and Perkins posited that the hallmark of high-road transfer is "mindful abstraction," or metacognition (Fuchs et al., 2003a). With metacognition, an