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Unconventional modelling of complex system via cellular automata and differential evolution

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ABSTRACT

The article deals with principles and utilization possibilities of cellular automata and differential evolution within task resolution and simulation of an epidemic process. The modelling of the spread of epidemics is one of the most widespread and commonly used areas of a modelling of complex systems. The origins of such complexity can be investigated through mathematical models termed 'cellular automata'. Cellular automata consist of many identical components, each simple, but together capable of complex behaviour. They are analysed both as discrete dynamical systems, and as information-processing systems. Cellular Automata (CA) are well known computational substrates for studying emergent collective behaviour, complexity, randomness and interaction between order and chaotic systems. For the purpose of the article, cellular automata and differential evolution are recognized as an intuitive modelling paradigm for complex systems. The proposed cellular automata supports to find rules of the transition function that represents the model of a studied epidemic. Search for models a studied epidemic belongs to inverse problems whose solution lies in a finding of local rules guaranteeing a desired global behaviour. The epidemic models have the control parameters and their setting significantly influences the behaviour of the models. One way how to get proper values of the control parameters is use evolutionary algorithms, especially differential evolution (DE). Simulations of illness lasting from one to ten days were performed using both described approaches. The aim of the paper is to show a course of simulations for different rules of the transition function and how to find a suitable model of a studied epidemic in the case of inverse problems using a sufficient amount of local rules of a transition function.

1. Introduction

Cellular Automata (CA) are well known computational substrates for studying emergent collective behaviour, complexity, randomness and interaction between order and chaotic systems. Cellular Automata are used for the simulation of complex phenomena, such as growth, reproduction, evolution or chemical reactions, etc. A complex organized behaviour is produced by interaction of many elements forming the monitored system. An understanding the processes, which form a natural complexity, allows us to understand the complexity in general and then these knowledge will be used for a proposal of software, which allows better understanding the behaviour of complex systems.

The first, who dealt with the issue of reproduction of a natural processes' complexity on a computer, was a Hungarian mathematician John von Neumann, who created the foundation for the study of cellular automata, which presents a natural way of complex systems' modelling on a computer [24]. In 1970 Conway created the game *Life* [1,7], where he designed a two-dimensional CA with such rules to avoid the formation of structures that quickly disappear or grow freely. In the seventies of last century the first deterministic two-dimensional CA was created to study statistical properties of a gas, the so-called HPP gas model [8]. This way has opened the possibility for simulation of motion, fluids, or granular substances using CA. In 1986, two-dimensional stochastic model FHP was proposed to study the movement of fluids [8], which reflects a realistic fluid dynamics. It was an undeniable proof of the ability of cellular automata to model real physical problems. In the 1980 s Stephen Wolfram studied cellular automata and dealt with the relationship between local rules and global behaviour. He studied the simplest one-dimensional automata and he divided CA according to long-term behaviour into four classes [24]. Stephan Wolfram is also the author of SW *Mathematica*, which allows a creation of simulation models using cellular automata [28].

The main advantages of simulation using CA include the ability to solve highly complex problems that are unsolvable by conventional analytical methods. CA also allows us the better understanding of modelled systems. The aim of this article is to use the

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proposed application to perform simulations of the spread of epidemics for different transition functions of CA, as well as the influence of changes in rules of the transition functions on the course of simulations is monitored. Results from our experimental study are compared with real data from 1978 flu epidemic.

Concerning differential evolution (DE), it has desired properties necessary to handle complex problems with interdependencies between input parameters, without the implementation complexity and computation cost [26]. Evolutionary operators used in differential evolution algorithm are very much suitable to tackle complex problems such as a complex system modelling, especially modelling the spread of epidemics. The epidemic models have the control parameters and their setting significantly influences the behaviour of the models. One way how to get proper values of the control parameters is use evolutionary algorithms, especially differential evolution [23].

2. Cellular automata

Cellular Automata (CA) are discrete, abstract computational systems that have proved useful both as general models of complexity and as more specific representations of non-linear dynamics in a variety of scientific fields [3]. First, CA are (typically) spatially and temporally discrete: they are composed of a finite or denumerable set of homogeneous, simple units, the atoms or cells. At each time unit, cells instantiate one of a finite set of states. They evolve in parallel at discrete time steps, following state update functions or dynamical transition rules: the update of a cell state obtains by taking into account the states of cells in its local neighbourhood (there are, therefore, no actions at a distance). Second, CA are abstract, as they can be specified in purely mathematical terms and implemented in physical structures. Third, CA are computational systems: they can compute functions and solve algorithmic problems. Despite functioning in a different way from traditional, Turing machine-like devices, CA with suitable rules can emulate a universal Turing machine, and therefore compute, given Turing's Thesis, anything computable.

All CA can be generated through tuning the four parameters that define their structure [22]:

a) *Discrete n-dimensional lattice of cells*: We can have one-dimensional, two-dimensional, …, n-dimensional CA. The atomic components of the lattice can be differently shaped: for example, a 2D lattice can be composed of triangles, squares, or hexagons. Usually homogeneity is assumed: all cells are qualitatively identical.
b) *Discrete states*: At each discrete time step, each cell is in one and only one state, $\sigma \in \Sigma$, Σ being a set of states having finite cardinality $|\Sigma|=k$.
c) *Local interactions*: Each cell's behaviour depends only on what happens within its local *neighbourhood* of cells (which may or may not include the cell itself). Lattices with the same basic topology may have different definitions of neighbourhood (the Moore neighbourhood, von Neumann neighbourhood, hexagonal neighbourhood, triangle neighbourhood etc.)
d) *Discrete dynamics*: At each time step, each cell updates its current state according to a deterministic transition function φ: $\Sigma^n \rightarrow \Sigma$ mapping neighbourhood configurations (n-tuples of states of Σ) to Σ. It is also usually, though not necessarily, assumed that (i) the update is *synchronous*, and (ii) φ takes as input at time step t the neighbourhood states at the immediately *previous* time step $t-1$.

The simplest class of one-dimensional cellular automata. Elementary cellular automata have two possible values for each cell (0 or 1), and rules that depend only on nearest neighbour values. As a result, the evolution of an elementary cellular automaton can completely be described by a table from Fig. 1 specifying the state a given cell will have in the next generation based on the value of the cell to its left, the value the cell itself, and the value of the cell to its right. Since there are 8 (2^3) possible binary states for the three cells neighbouring a given cell, there are a total of 256 (2^8) elementary cellular automata, each of which can be indexed with an 8-bit binary number [28]. Starting with random initial conditions, Wolfram went on to observe the behaviour of each rule in many simulations. As a result, he was able to classify the qualitative behaviour of each rule in one of four distinct classes, which are the following:

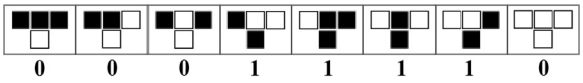

Fig. 1. A table specifying the state a given cell based on the value of the cell to its left, the value the cell itself, and the value of the cell to its right.

- Class1—rules leading to homogenous states, all cells stably ending up with the same value.
- Class2—rules leading to stable structures or simple periodic patterns.
- Class3—rules leading to seemingly chaotic, non-periodic behaviour.
- Class4—rules leading to complex patterns and structures propagating locally in the lattice.

CA are systems showing properties of self-organization, but their ability to distinguish status of environments, which are important for its maintenance, is low. As an example, we can mention a growth of solid crystals or the aforesaid oscillating chemical reactions. These systems are either almost unresponsive to changes in the environment due to their regular and fixed internal structure (crystals), or they are highly sensitive to environmental changes (e.g. amount of supplied energy) because their internal structure shows a high degree of disorder (oscillating reaction). Differences between these two extreme examples may be illustrated by appropriate mathematical models of self-organization [19]. One of the most popular, easiest as well as the most illustrative models are primarily cellular automata.

The utilization of CA consists in their displaying complex emergent behaviour, starting from simple atoms deterministically following simple local rules. Because of this, CA attract a growing number of researchers willing to study pattern formation and complexity in a pure, abstract setting. In his review of the literature, Andrew Ilachinski [22] narrows down CA applications to four main areas:

- Cellular automata as powerful computational engines.
- Cellular automata as discrete dynamical system simulators.
- Cellular automata as conceptual vehicles for studying pattern formation and complexity.
- Cellular automata as original models of fundamental physics.

Concerning cellular automata as discrete dynamical system simulators, the area comprises scientific applications of CA to the modelling of specific problems—to mention just a few: urban evolution [2], Ising models [9], neural networks [12], turbulence phenomena [6] etc.