Swarm and Evolutionary Computation 19 (2014) 1–14

Contents lists available at ScienceDirect

Swarm and Evolutionary Computation

journal homepage: www.elsevier.com/locate/swevo

Regular Paper

Metaheuristic multi-objective optimization of constrained futures portfolios for effective risk management

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A R T I C L E I N F O

Article history:
Received 28 March 2014
Received in revised form
17 July 2014
Accepted 9 August 2014
Available online 27 August 2014

Keywords:
Multi-objective optimization
Evolution strategy
Differential evolution
Futures markets
Diversification
Risk budgeting

A B S T R A C T

In the Derivatives financial markets, Futures portfolios are perceived to be instruments of high risk, despite their flexibility of being used for portfolio protection (hedging) or for profitable trading (speculating). A multi-pronged approach for an effective management of the risks involved includes employing strategies such as, diversification between dissimilar markets, decision to go long or short on assets that make up the portfolio and risk tolerance or risk budgeting concerned with how risk is distributed across asset classes constituting the portfolio with all of these governed by investors' preferences and capital budgets. However, the inclusion of such objectives and constraints turns the problem model complex for direct solving using analytical methods, inducing the need to look for metaheuristic solutions.

In this paper, we present a metaheuristic solution to such a complex futures portfolio optimization problem, which strives to obtain an optimal well-diversified futures portfolio combining several asset classes such as equity indices, bonds and currencies, subject to the constraints of risk and capital budgets imposed on each of the asset classes, besides bounding constraints. The Herfindahl index function has been adopted to measure diversification of the long-short portfolio. In the absence of related work and considering the complexity of the problem that transforms it into a non linear multi-objective constrained optimization problem model, two metaheuristic strategies viz., multi-objective evolution strategy and multi-objective differential evolution, chosen from two different genres of evolutionary computation, have been employed to solve the complex problem and compare the results. Extensive simulations including performance analyses, convergence testing and back testing portfolio reliabilities have been undertaken to analyze the robustness of the optimization strategies.

1. Introduction

Futures contracts are key players in the *Derivative financial markets*, which include a variety of other financial contracts such as Options, Forwards and Swaps, and variations of these [1]. A futures contract is an agreement to buy or sell a specified amount of commodity or a non-commodity such as index, currency, bond or other asset of value, for a certain price at a certain time in the future. The contract sets the value and is usually referred to by its delivery month which is set ahead of time. The price is known as the *futures price*. The futures contracts can be bought or sold only on the *futures exchange*. Anyone who *buys a futures contract* directly or through one of the pooled investment alternatives is said to take up a *long position* and anyone who *sells a futures contract* by closing the position with a closing purchase transaction is said to take up a *short position*. Thus one of the crucial elements of the futures contracts is the decision on which positions need to be taken long or short which defines and distinguishes risk as well. Thus, futures markets deal with risk and risk transfer where investors (*hedgers*) hedge positions against risk by transferring the same to investors (*speculators*) who are willing to accept those risks in exchange for the profit potential. The profits and losses of a futures contract depend on the daily movements of the market for that contract and are calculated on a daily basis.

However, earning profits through futures trading is fraught with risks and therefore a perspective on futures requires a complete appreciation of the risks involved [2]. *Diversification* and the process of *allocation* which involves investments in different markets is a popular strategy to mitigate risk. With different securities performing differently at any point in time, the objective of diversification is to minimize financial losses by constructing a portfolio of mixed asset classes and types so that the decline of one or more assets in the portfolio does not severely

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http://dx.doi.org/10.1016/j.swevo.2014.08.002
2210-6502/

impact the portfolio. Asset allocation based on *Capital budgeting,* which involves how much money is to be invested on different assets comprising the portfolio is natural. But considering the risks involved, it is more than essential to adopt asset allocation based on *Risk budgeting* too, which focuses on how risk is distributed over assets or asset classes throughout the portfolio. In this aspect, a clear understanding of the investors' risk profiles which involves an analysis of the investors' risk levels as well as personal risk tolerance (*risk budgets*) and investing objectives, are also essential to safely manage risk. Since both long future positions and short future positions are *unlimited risk- unlimited returns* positions that can be entered into, by futures speculators to profit from the rise or fall of the underlying positions respectively, the long-short mix of the futures portfolio also plays a dominant role in managing one's risk. An investor may exercise a choice for a *fully invested portfolio* where virtually all funds stay invested over the portfolio with no cash reserve in the account or even *more than fully invested* when he or she may *margin* or *borrow* funds for investment. An investor who is fully invested or more than fully invested runs the risk of severe financial loss if the markets do not perform as expected. All of these strategies needless to say, are subject to the investors' preferences and capital budget requirements over different assets/asset classes comprising the portfolio.

The objective of this work, therefore is to obtain an *optimal futures portfolio* that is well-diversified, fully invested, and accommodates a long-short position mix, with risk budgets and asset class constraints imposed on the portfolio positions to reflect the risk profiles and investment objectives of the investor. Viewed under the Markowitz framework [3] where *variance of a portfolio* models risk, the main focus of the work is on obtaining a *minimum variance futures portfolio* or a minimum risk futures portfolio, subject to the constraints imposed and hence the work precludes inclusion of futures returns in its present exploration.

An optimal futures portfolio thus strives to enlighten the investor on what proportion of funds should be invested on what asset—termed *weights* in portfolio management parlance. The decision variables of the futures portfolio optimization problem are thus the weights $(W_1, W_2, W_3 \ldots W_N)$, where N is the number of assets in the portfolio. In the case of long positions the weights are deemed positive and in the case of short positions they are deemed negative. The objective of the optimization problem is to maximize diversification which notionally serves to minimize risk. Entropy based diversification has been a popular measure to quantify risk, in a long-only portfolio though. But Woerheide and Persson [4] recommended *Herfindahl index* as the best measure of diversification for an unevenly distributed portfolio, out beating even entropy based diversification. Therefore this work chose to employ Herfindahl index based diversification. However, the Herfindahl index by and large, has been applied only over *long-only portfolios* (portfolios with only long positions, that is positive weights) in the literature. In this work considering the long-short mix of the portfolio (that is, positive and negative weights) – a strategy that had to be executed to mitigate risk – a revised definition of Herfindahl index has been proposed and applied for maximizing diversification. The investor preference for a fully invested portfolio represents itself as a constraint where sum of the weights equals 1. The long-short portfolio mix is represented as a bounding constraint where each weight of the portfolio falls within a range $[-a, b]$. The capital budget requirements of the investor are represented as constraints where the sum total of the weights of the assets in each of the asset classes (for example, equity indices, bonds and currencies—in this work) lie between bounds specified by the investor. Finally the risk budgets specified as x% of the portfolio risk for each of the asset classes, is represented as a non-linear constraint involving the variance–covariance matrix of the assets in the asset classes concerned.

To tackle the complexity of the problem objectives and constraints, the mathematical formulation of the problem model transforms itself into a non-linear, multi-objective, constrained portfolio optimization problem, which renders it difficult for direct solving using analytical methods. Multi-objective evolutionary algorithms have presented themselves as ideal candidates to tackle such complex computationally expensive optimization problems due to their inherent nature of managing multiple conflicting objectives, delivery of Pareto optimal solutions in a single run and malleability to accommodate extensions and refinements of their algorithmic frameworks and/or evolutionary operators, to facilitate efficient problem solving [5]. Hence two metaheuristic strategies viz., *multi-objective evolution strategy* and *multi-objective differential evolution*, both belonging to two different genres of evolutionary computation and refined versions themselves of their conventional counterparts to tackle multi-objective optimization, have been employed to solve the problem and compare the results.

A survey of literature revealed that very little work had been reported in the area of constrained optimization for a futures portfolio of this nature. Putzig, Becherer and Horenko [6] discussed a futures portfolio optimization problem that dealt with a long–short portfolio for commodities with basic constraints and which employed a numerical optimization strategy based on the Tykhonov-type regularization for its solution. You and Daigler [7] examined the diversification benefits of using individual futures contracts instead of simply a commodity index. Benth and Lempa [8] explored futures portfolio optimization from the point of view of maximizing utility from the final wealth when investing in futures contracts.

However, the work discussed in this paper is distinct from its peers in that, the objectives of diversification and risk budgeting which yield non-linear constraints and hence demanded metaheuristic optimization strategies for its solution, and capital budgeting manifesting itself as asset class constraints, have been hitherto remained unexplored in the futures market scenario. In fact it was the absence of such counterparts against which the results could be compared, that compelled investigation of two metaheuristic strategies chosen from two different genres of evolutionary computation, to ascertain the robustness and accuracy of the solutions to the complex constrained optimization problem through comparison.

Section 2 discusses the mathematical formulation of the non-linear constrained futures portfolio optimization problem. Section 3 briefly discusses the two metaheuristic strategies of evolution strategy and differential evolution. Section 4 outlines the application of the metaheuristic methods for the solution of the futures portfolio optimization problem. Section 5 details the experimental results to study the performance and analyse the robustness of the solution strategies and Section 6 lists the conclusions of the study.

2. Mathematical formulation

Let N be the number of assets in the futures portfolio, $\mathbf{W} = (W_1, W_2, W_3 \ldots W_N)$ the *weights* or the proportion of capital to be invested in the assets and σ_{ij} the covariance between the daily returns of assets i and j.

The *portfolio risk* σ_P following Markowitz's framework [3] is given by

$$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} W_i W_j \sigma_{ij} \tag{1}$$

The *Annualized Portfolio Risk* is defined as $\sqrt{261} \times \sigma_P$ where 261 is generally the number of trading days in a year.