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State estimation of nonlinear stochastic systems using a novel meta-heuristic particle filter

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ABSTRACT

This paper proposes a new version of the particle filtering (PF) algorithm based on the invasive weed optimization (IWO) method. The sub-optimality of the sampling step in the PF algorithm is prone to estimation errors. In order to avert such approximation errors, this paper suggests applying the IWO algorithm by translating the sampling step into a nonlinear optimization problem. By introducing an appropriate fitness function, the optimization problem is properly treated. The validity of the proposed method is evaluated against three distinct examples: the stochastic volatility estimation problem in finance, the severely nonlinear waste water sludge treatment plant, and the benchmark target tracking on re-entry problem. By simulation analysis and evaluation, it is verified that applying the suggested IWO enhanced PF algorithm (PFIWO) would contribute to significant estimation performance improvements.

1. Introduction

State estimation plays a key role in different applications such as fault detection, process monitoring, process optimization, and model based control techniques [1]. Fortunately, a large group of models in signal processing can be represented by a state-space form in which prior knowledge of the system is available. This prior knowledge allows us to exploit a Bayesian estimation approach. Within this statistical framework, one can perform inference on the unknown states according to the posterior distribution. In most cases, the observations arrive sequentially in time, and one is interested in recursively estimating the *hidden* states from the time-varying posterior distribution. This problem is referred as the *optimal filtering* problem [2,3]. Owing to the mathematical complexity, only few specific models (including linear Gaussian state-space models and finite state-space hidden Markov models (HMM) [4]) can be adopted to reach an analytical solution. The popular Kalman filter (KF) [2,3] and the renowned HMM filter [4] provide close form solutions to the latter models.

In many real-life applications, however, the models possess nonlinearity and non-Gaussian behavior. Thus, an optimal solution to the filtering problem cannot be attained. In this case, it becomes necessary to exploit approximate and computationally traceable sub-optimal solutions to the sequential Bayesian estimation methodology. Over the past decades, several sub-optimal filtering methods such as the extended Kalman filter (EKF), and the unscented Kalman filter (UKF) have been proposed in the open literature [5]. But, these filtering algorithms suffer from the *curse of dimensionality*; that is, they perform poorly as the dimension of the model states increases. Furthermore, the rate of convergence of the approximation error decreases dramatically for large state dimensions, say 4 [5]. Notably, it has been demonstrated that the estimation performance of UKF inhibits intrinsic limitations. In other words, the deterministic choice of the so called *sigma points* confines the flexibility desired to construct a probability distribution.

The particle filter (PF), first brought forward by Gordon et al. [6], employs a set of N random samples (or particles) to approximate the posterior distribution. The particles are evolved over time via a combination of importance sampling and re-sampling steps. In a few words, the re-sampling step statistically multiplies and/or discards particles at each time step to adaptively concentrate particles in the regions of high posterior probability [7]. The popularity of the PF results from the notion that it does not call for model simplification or adopting special distributions.

Recently, researchers have shown an increased interest in the subject of integrating meta-heuristic algorithms in PF. In a seminal paper, Tong et al. [8] proposed an optimized PF based on particle swarm optimization (PSO) algorithm [9] which demonstrated improved estimation accuracy. Many subsequent studies also followed the same trend using PSO; e.g., refer

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to [10,11]. In [10], the authors exploited a similar method based on PSO for visual tracking, and claimed that the modified scheme has better accuracy than the conventional PF. Later, Jing et al. [11] further advanced the algorithm brought forward in [8] with a new re-sampling strategy. However, to authors' knowledge, there has been very little discussion on developing other meta-heuristic based PF algorithms, thus far. The studies reported to date have focused on adjusting the PSO enhanced PF algorithm rather than incorporating other techniques established upon evolutionary algorithms and swarm intelligence.

The bio-inspired IWO algorithm was introduced by Mehrabian and Lucas [12] which imitates the colonial behavior of invasive weeds in nature. The IWO algorithm has shown to be virtuous in converging to optimal solution by employing some basic characteristics of weed colonization, e.g. seeding, growth and competition. In [13], Chakraborty et al. investigated the search performance and specifically the effect of population variance on the explorative power of the algorithm. Later, Roy et al. [14] proposed a hybrid optimization algorithm by integrating the optimal foraging theory in IWO which evinced improved optimization capacity. Previously, the IWO algorithm has been utilized in a surfeit of applications including optimizing and tuning of a robust controller [12], antenna configuration optimization [15], optimal arrangement of piezoelectric actuators on smart structures [16], DNA computing [17], and etc.

This paper considers the implementation of the IWO algorithm as a mean to optimize the PF method. Since sampling in PF is carried out in a sub-optimal manner, it can bring about some performance defects such as *sample impoverishment* [5]. By introducing a suitable fitness function for particles, such problems are circumvented and an enhanced PF algorithm is achieved thanks to the IWO approach. The functionality of the combined method is verified using three nonlinear state estimation problems from different fields: volatility estimation of a stock market, state estimation of a nonlinear chemical process, and the re-entry vehicle tracking problem.

The rest of this paper is organized as follows. Section 2 provides a concise description of some preliminary notions including the filtering problem, the Monte Carlo method, Importance Sampling, and the basic particle filtering algorithm. The IWO algorithm is limned in Section 3. The proposed PFIWO method is discussed in Section 4. Simulation Results based on the PFIWO algorithm and some discussions are outlined in Section 5. Section 6 concludes the paper.

2. Preliminaries

2.1. The filtering problem

Consider the general class of nonlinear non-Gaussian systems with state-space model as described below

$$x_k = f(x_{k-1}, u_{k-1}, v_{k-1}), \quad x_k \sim p(x_k|x_{k-1}) \tag{1a}$$

$$y_k = g(x_k, u_k, w_k), \quad y_k \sim p(y_k|x_k), \tag{1b}$$

where the subscript k denotes the time instance. $x_k \in R^{n_x}$ represent the system states with probability distribution of $p(x_k|x_{k-1})$ which is not directly measurable, and $y_k \in R^{n_y}$ is the noise corrupted observation with likelihood $p(y_k|x_k)$. The maps $f \in R^{n_x} \times R^{n_u} \times R^{n_v} \rightarrow R^{n_x}$ and $g \in R^{n_x} \times R^{n_u} \times R^{n_w} \rightarrow R^{n_y}$ are generally nonlinear functions. u stands for known inputs. v and w represent the process and measurement noise, respectively. The overall structure is illustrated in Fig. 1. *Filtering* is the task of sequentially estimating the states (parameters or hidden variables) of a system as a set of observations become available on-line [2,3]. Strictly speaking, filtering is aimed at estimating the posterior distribution $p(x_k|y_k)$ *as* a set of observations $Y_k = (y_1, y_2, \ldots, y_k)^T$ arrives. It is worth noting that the results obtained in this section are established upon the following assumptions:

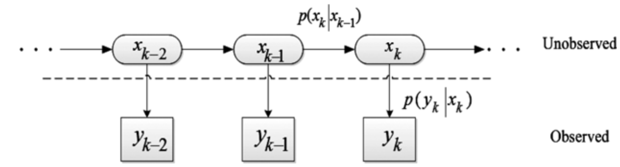

Fig. 1. A graphical representation of the state-space model described by Eq. (1).

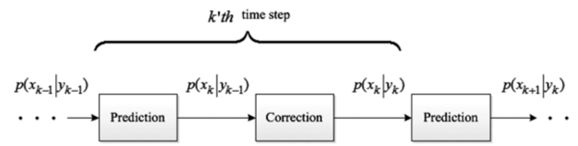

Fig. 2. The Bayesian approach to the filtering problem.

1. The states follow a first order Markov process, i.e., $x_k|x_{k-1} \sim p_{x_k|x_{k-1}}(x_k|x_{k-1})$ with an initial distribution of $p(x_0)$.
2. The measurements are conditionally independent given the states, i.e., each y_k only depends on x_k.

The Bayesian solution to the filtering problem consists of two stages [2,3,5]:

1. Prediction: let the above assumptions hold. Using the prior density function, and the Chapman–Kolmogorov equation we have

$$p(x_k|y_{k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|y_{k-1})dx_{k-1}. \tag{2}$$

2. Correction: based on the Bayes' formula

$$p(x_k|y_k) = \frac{p(y_k|x_k)p(x_k|y_{k-1})}{p(y_k|y_{k-1})} \tag{3a}$$

wherein

$$p(y_k|y_{k-1}) = \int p(y_k|x_k)p(x_k|Y_{k-1})dx_k. \tag{3b}$$

The algorithm is initialized with $p(x_0|y_0) = p(x_0)$ and $p(x_1|y_0) = p(x_1)$. One step operation of the Bayesian filtering is portrayed in Fig. 2. However, it is obvious that achieving a closed form analytical solution to the untraceable integral in Eq. (2) and therefore the solution to Eq. (3) is a cumbersome task. The problem becomes even more severe as the state dimensions increase. Thus, an optimal solution cannot be attained except under very restricting conditions (linear transition functions and Gaussian noise) using the well-known KF. The interested reader can refer to [2,3] which provide a comprehensive theoretical overview of available optimal methods. Sub-optimal solutions exist for rather general models with nonlinear evolution functions and non-Gaussian noises. Nevertheless, due to the nature of these methods (e.g. EKF and UKF) which are based on local linearization, the estimation performance is, more or less, limited. Estimation techniques established upon sequential Monte Carlo methods, namely the PF, are a promising alternative to local linearization algorithms [6,18].

2.2. Monte Carlo and importance sampling techniques

In the Monte Carlo technique, one is concerned with estimating the properties of some highly complex probability distribution $p(x)$, e.g. *expectation*

$$E(s(x)) = \int s(x)p(x)dx \tag{4}$$

where $s(x)$ is some useful function for estimation. In cases where this cannot be obtained analytically, the approximation problem can be handled indirectly. It is possible to represent $p(x)$ by a set