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The natural number bias and its role in rational number understanding in children with dyscalculia. Delay or deficit?

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A R T I C L E I N F O

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A B S T R A C T

Background: Previous research indicated that in several cases learners' errors on rational number tasks can be attributed to learners' tendency to (wrongly) apply natural number properties. There exists a large body of literature both on learners' struggle with understanding the rational number system and on the role of the natural number bias in this struggle. However, little is known about this phenomenon in learners with dyscalculia.
Aims: We investigated the rational number understanding of learners with dyscalculia and compared it with the rational number understanding of learners without dyscalculia.
Method: Three groups of learners were included: sixth graders with dyscalculia, a chronological age match group, and an ability match group.
Results: The results showed that the rational number understanding of learners with dyscalculia is significantly lower than that of typically developing peers, but not significantly different from younger learners, even after statistically controlling for mathematics achievement.
Conclusion: Next to a delay in their mathematics achievement, learners with dyscalculia seem to have an extra delay in their rational number understanding, compared with peers. This is especially the case in those rational number tasks where one has to inhibit natural number knowledge to come to the right answer.

What this paper adds

While there exists a large body of literature both on learners' struggle with understanding the rational number domain and on the role of the natural number bias in this struggle, little is known about the rational number understanding of learners with dyscalculia (LWD). Nonetheless, a better insight of the understanding of rational numbers of LWD is important to provide adaptive (remedial) instruction with the aim to increase LWD's understanding of the rational number system. Moreover, the studies that did already investigate LWD's rational number understanding only focused on their understanding of the magnitude of rational numbers. The present study includes a wider range of problem types, including both magnitude, operations, and density tasks, enabling us to provide a more complete picture of the rational number understanding. The results of the present study add insight in the rational number understanding of LWD by suggesting that LWD have an additional delay on their rational number understanding. While this is the case for all types of rational number tasks, the difference in accuracy level between LWD and their peers is much higher in those rational number tasks where natural number reasoning leads to an incorrect answer. So, this study suggests that LWD are more affected by the natural number bias compared to their peers, leading to even more difficulties understanding the rational number

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system.

1. Introduction

Rational numbers are known to form a stumbling block for many people. Not only elementary school children, but also secondary school students, adults, and even (prospective) teachers have trouble understanding the rational number domain (e.g., Cramer, Post, & delMas, 2002; Mazzocco & Devlin, 2008; Vamvakoussi, Van Dooren, & Verschaffel, 2012; Vamvakoussi & Vosniadou, 2004). For instance, in a recent study of Van Hoof, Verschaffel, and Van Dooren (2015), only 57% of a large group of eighth graders solved the question "What is half of 1/8?" correctly. The most frequent erroneous answer was 1/4. It is quite worrying that so many learners have troubles understanding rational numbers, some of which even last until the end of secondary school. This is especially worrying since the understanding of rational numbers has been shown to relate to later mathematics achievement in general and for performance in important domains of the mathematics curriculum in particular such as algebra (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012; Siegler, Thompson, & Schneider, 2011). For example, Booth and Newton (2012) found that learners' understanding of fractions is a better predictor of learners' algebra readiness than their whole number magnitude knowledge.

1.1. The natural number bias

There are several distinct reasons why learners are found to struggle so much with the understanding of rational numbers (e.g., Van Hoof, Vamvakoussi, Van Dooren, & Verschaffel, 2017). For example, learners need to understand the several distinct conceptual meanings of fractions such as fractions as parts of a whole, fractions as ratios, fractions as numbers (Lamon, 2001) and the fact that different representations can represent the same magnitude (e.g., $\frac{1}{2}$, $\frac{3}{6}$,. 0.5, 0.50, 50%) (Moss, 2005). Siegler, Fazio, Bailey, and Zhou (2013) discuss the possible confusion when conducting arithmetic operations with rational numbers. They give the example that adding and subtracting fractions with the same denominator will result in a fraction with the same denominator, whereas this is not the case for multiplication and division. This leads to mistakes such as "$2/5 \times 3/5 = 6/5$".

One frequently recurring explanation for learners' difficulty to understand the rational number system is the natural number bias, a phenomenon that has generated substantial research interest both in the fields of mathematics education and cognitive psychology (Jordan et al., 2013; Merenluoto & Lehtinen, 2004; Ni & Zhou, 2005; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013; Vamvakoussi, Christou, Mertens, & Van Dooren, 2011; Vamvakoussi, Van Dooren, & Verschaffel, 2012; 2013; Van Hoof, Janssen, Verschaffel, & Van Dooren, 2015; Van Hoof, Lijnen, Verschaffel, & Van Dooren, 2013; Van Hoof, Vandewalle, Verschaffel, & Van Dooren, 2015). Ni and Zhou (2005) define the natural number bias as the tendency to apply natural number properties in tasks with rational numbers, even when this is inappropriate. The phenomenon is described as follows: the rational number system is the first new number system learners encounter after the natural number system. Therefore, to comprehend the rational number system, learners have to construct an entirely new number concept and new procedures to handle numbers, which are not always in accordance with their prior knowledge of (natural) numbers. In line with this description, numerous studies indeed indicated that learners make systematic and predictable errors in rational numbers tasks where the use of prior natural number knowledge leads to the incorrect answer (further referred to as inconsistent items) while at the same time they are much more accurate in rational number tasks where reliance on prior natural number knowledge leads to the correct answer (further referred to as consistent items) (Vamvakoussi et al., 2012; Van Hoof, Vandewalle et al., 2015).

Previous research on the natural number bias mainly focused on three aspects in which rational numbers differ from natural numbers and where errors are known to occur: their dense structure, the way their numerical magnitude can be determined, and the effect of the four basic operations.

Regarding the first aspect, rational numbers are dense. Contrary to natural numbers, rational numbers do not obey the successor principle. While with natural numbers one can always point out which number follows a given number, it is impossible to say which number follows a given rational number, because between any two given rational numbers are always infinitely many other numbers. Several studies report the struggle learners have with understanding the dense structure of rational numbers (Merenluoto & Lehtinen, 2004, Vamvakoussi & Vosniadou, 2004, 2010).

Concerning the second aspect, the numerical magnitude of rational numbers, you cannot always rely on the numerical value of the natural numbers involved to determine the magnitude of rational numbers. With fractions, it is not always true that a fraction's numerical value increases when its denominator, numerator or both increase. In the same line, with decimal numbers, longer decimals are not always larger and shorter decimals are not always smaller. These differences between rational and natural numbers in the way their numerical magnitude can be determined are known to lead to many mistakes by learners (Christou & Vosniadou, 2012; Resnick et al., 1989).

Regarding the last aspect, the effect of arithmetical operations, with rational numbers, multiplication and addition do not always lead to a larger outcome and division and subtraction do not always lead to a smaller outcome, while this is the case with natural numbers. Several studies indicated that learners still rely on the natural number rules while solving rational number tasks, leading to systematic errors (Vamvakoussi et al., 2012; Van Hoof, Vandewalle et al., 2015; Christou & Vosniadou, 2012).

In sum, these three aspects on which natural numbers differ from rational numbers are reported to lead to many misconceptions about rational numbers, which lead in their turn to those systematic errors in inconsistent rational number tasks.