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Taiwanese high school students' perspectives on effective mathematics teaching behaviors

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ABSTRACT

This study developed a questionnaire composed of teaching-behavior items obtained from pioneering empirical studies, for measuring students' perspectives on effective mathematics teaching. The questionnaire was used to survey 4514 high school students in Taiwan, where Western innovative mathematics instruction has been promoted since a curriculum reform in 1993. Exploratory factor analysis revealed factors contributing to how teachers should handle teaching materials, use representations and teaching methods, and guide problem solving. Ten factors were identified. Seven are rooted in traditional Chinese educational culture (e.g., *detailed illustration* in teaching problem solving), and three were influenced by Western culture (e.g., *student active-learning activities* in teaching methods). The findings show that teaching behaviors that either require a considerable amount of time or minimize use of time are relatively less favored, including learning in small groups or through games in the *student active-learning activities* factor and only providing important steps in the *speedy lecture* factor.

1. Introduction

Countries in East Asia have been outperforming their Western counterparts in mathematics achievements in several international comparative studies such as the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (e.g., Mullis, Martin, & Foy 2008; Mullis et al., 2015; Organization for Economic Cooperation and Development, 2016). The findings suggested the need for research on effective or good mathematical instruction in various countries. In Eastern countries, for example, Pang (2009) conducted a case study to analyze a sixth-grade teacher's instructional approach to investigate the characteristics of good mathematics teaching in Korea. Kaur (2009) explored teachers' practices and students' perspectives to determine what constitutes good mathematics teaching at the junior high school level in Singapore. In Western countries, Murray (2011) investigated senior high school students' opinions on good mathematics teaching in Australia, as did Martinez-Sierra (2014) in Mexico. Teachers' perspectives on effective mathematics teaching have been explored in China (Wang & Cai 2007a), Hong Kong (Wong, 2007), Australia (Perry, 2007), and the United States (Wang & Cai 2007b), and comparative research has investigated views from Eastern and Western cultures (Bryan et al., 2007). A comparison with the addition of European perspective was also conducted to elucidate effective mathematics teaching (Kaiser & Vollstedt 2007). In the aforementioned studies, a critical criterion for judging effective teaching is whether the instruction given to students in the classroom could cultivate their understanding of mathematics. This has been strongly supported by prevalent opinion in mathematics teaching and learning for decades (Llewellyn 2012; National Council of Teachers of Mathematics, 1989). Therefore, it was adopted in the present study.

The literature provides an overview of effective mathematics teaching in various cultural contexts, including the commonality and uniqueness of teaching approaches used in different cultures. The judgement of effective mathematics teaching behavior is culturally value-loaded. Some features that characterize good teaching in East Asian cultures are not favored in Western cultures (Li 2011; Pang, 2009) and possibly the converse is also true. Therefore, we believe that studying effective mathematics teaching in Taiwan will contribute to international views on this issue, not only because international comparative research reports Taiwan to be among the highest-achieving countries in mathematics, but also because two major mathematics instructional perspectives have existed in Taiwan since the country's curriculum reform in 1993 (Hsieh, 1997). One perspective represents traditional approaches that involve heavy teacher control; the other perspective represents approaches adopted from the West, involving student-

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oriented and constructivist-based instruction (Hsieh, 1997; Hsieh et al., 2015).

Effective teaching has been explored for decades (e.g., McLeish, 1978); until now, various research methods, such as observing teaching practices in the classroom and investigating the perspectives of teachers and students through interviews and questionnaires, have been employed in numerous studies, including studies on effective teaching specifically in the context of mathematics (e.g., Johnson, 1997; Lin & Li 2009; Liu & Meng 2009). Methods for investigating students' opinions inform researchers and educators about the needs of students and thus particularly reflect the student-centered view (Alhija, 2016; Kane & Maw 2005; Spooren & Christiaens 2016), which has been promoted in innovative mathematics instruction since Taiwan's curriculum reform (Hsieh, 1997). Furthermore, because the teacher-directed view is embedded in traditional East Asian culture, the power between students and teachers is unbalanced in Taiwan. High school students have no opportunities to evaluate their mathematics teachers, and thus teachers have no opportunities to improve their mathematics teaching through listening to students' opinions. To investigate students' perspectives, questionnaire surveys are an efficient method for collecting large-scale cross-sectional data. However, in contrast to studies exploring the characteristics of effective teaching in a general sense (i.e., not in the context of a particular subject), in which several versions of questionnaires composed of closed-ended items containing broad dimensions have been developed (e.g., Liu, Xu, & Stronge 2016; Meng & Muñoz 2016; Muñoz, Scoskie, & French 2013), few similar questionnaires specific to the subject of mathematics are available.

Accordingly, the purposes of the present study are as follows:

1. Develop a questionnaire composed of teaching-behavior items for measuring students' perspectives on effective mathematics teaching;
2. Explore the factors contributing to effective mathematics teaching from the perspectives of Taiwanese high school students; and
3. Investigate Taiwanese high school students' endorsement of the factors (obtained from the second research purpose) and the differences in the perspectives between students at different school levels.

2. Conceptual framework

2.1. Understanding in mathematics

Researchers in general education have published various identifications of what understanding is; such ideas can be applied to mathematics education. Piaget (1936) notably identified understanding as a progressive reorganization of mental structure to integrate what one already knows to what one newly discovers. This identification bears the idea of an integrated and functional grasp of ideas by connecting what is to be learned with what already has been learned (in psychology, see Bransford, Brown, & Cocking 1999; in mathematics, see Kilpatrick et al., 2001).

One can judge the effectiveness of mathematics teaching by considering whether it helps students to understand mathematics; however, a problem encountered by the present study was to consider the nature of the content to be understood by the students. The mathematical content in the studies exploring students' mathematical understanding can be classified into two major categories: mathematical concepts and ideas and the solutions to problems (Jones, Jones, & Vermette 2009; Usiskin, 2012); this aligns with the emphasis in East Asian mathematics classrooms on the "two basics" (Leung, 2001), which are mathematical concepts and ideas (basic knowledge) and procedures or skills in dealing with them (basic skills). The first basic, mathematical concepts and ideas, is considered as the foundation, reflecting the distinctive knowledge structure of mathematics. One primary goal of mathematics learning is to get hold of this knowledge structure. This idea aligns with the critical component of mathematics proficiency proposed by Kilpatrick et al. (2001) — *conceptual understanding*, which they referred as a grasp of a coherent and interrelated whole composed of mathematical concepts, facts, properties, operations, principles, procedures, methods, and relations and connections among them. The present study adopted this perspective. The second basic, procedures and skills for dealing with mathematical concepts and ideas, involves not only accurate calculation and manipulation but also reasoning in solution processes according to logical rules, expressing them in a clear and formal way, and familiarity with solution patterns (Zhang, Li, & Tang 2004). This pertains to *procedural fluency*, *strategic competence*, and *adaptive reasoning*, which are also components of mathematical proficiency proposed by Kilpatrick et al., but in a narrower sense. To cultivate mastery of the second basic, grouped-items training is often employed by teachers in East Asia including Taiwan (Jiansheng, 2004). Taiwanese teachers usually teach a series of typical sample problems (Shao et al., 2013); they demonstrate how to solve one or several typical sample problems before giving students similar problems to practice. The organization of content in Taiwanese mathematics textbooks also reflects this teaching approach. The typical sample problems here are in accordance with Schoenfeld's (1992) description of the traditional use of the term "problems", which indicate the routine problems serve as vehicles of instruction to let students acquire and master mathematics skills and knowledge. The difficulty level of these problems can vary from basic ones to difficult and complex ones, and they are of various forms such as simple application of mathematical procedures or word problems. In the present study, the solutions that students are expected to understand are the solutions to typical sample problems taught in class.

In the present study, effective teaching that fosters students' understanding of mathematics involves understanding mathematical concepts and ideas and understanding problem solutions.

2.2. Teaching that could promote students' understanding of mathematics

2.2.1. Teaching that could help students understand mathematical concepts and ideas

In a review by Hiebert and Carpenter (1992), studies on teaching approaches that facilitate student mathematical understanding typically involve three concepts: handling teaching materials, representations, and teaching methods. These three concepts were also critical issues for studying effective pedagogy for teaching mathematical concepts (e.g., Jones et al., 2009; Kaur, 2009; Li, 2011; Van Es & Conroy 2009; Wilson et al., 2005) and were adopted by the present study. Handling teaching materials mainly reflects mathematical features of instruction; teaching methods mainly reflect pedagogical features; and representational tools reflect both features. Representations that teachers use in their instruction are both mathematical content for students to learn and teaching practices that support students' learning.

Regarding the methods for handling teaching materials to facilitate student understanding with mathematical concepts and ideas, one category of methods that has been frequently mentioned in related research involves teachers' assisting students in forming internal connections within mathematics and external connections between mathematics and other subjects or real-world (Ausubel, 1968; Davis, 2001; Jones et al., 2009). The other category of methods was based on the idea of Skemp (1971): "concepts of a higher order than those which a person already has cannot be communicated to him by a definition, but only by arranging for him to encounter a suitable collection of examples" (p. 32). The practice of arranging sufficient appropriate examples by which students can understand a mathematical concept through abstraction is considered an effective teaching behavior (Meel, 2003).

Representations in mathematics are of various types, such as physical objects, real-world examples, manipulatives, graphs, diagrams, tables, and symbols, and the levels of representations introduced become more and more abstract during the school years (Kilpatrick et al., 2001). According to Bruner (1966), representations can be classified into three main types: enactive, iconic, and symbolic representations;