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Strategy-proof school choice mechanisms with minimum quotas and initial endowments ☆

Naoto Hamada [a], Chia-Ling Hsu [b], Ryoji Kurata [a], Takamasa Suzuki [c,*], Suguru Ueda [d], Makoto Yokoo [a]

[a] Graduate School of Information Science and Electrical Engineering, Kyushu University, Fukuoka, Japan
[b] School of Economics, Southwestern University of Finance and Economics, Chengdu, China
[c] Faculty of Economics and Information, Gifu Shotoku Gakuen University, Gifu, Japan
[d] Department of Information Science, Saga University, Saga, Japan

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ABSTRACT

We consider a school choice program where minimum quotas are imposed for each school, i.e., a school must be assigned at least a certain number of students to operate. We require that the obtained matching must respect the initial endowments, i.e., each student must be assigned to a school that is at least as good as her initial endowment school. Although minimum quotas are relevant in school choice programs and strategy-proofness is important to many policymakers, few existing mechanisms simultaneously achieve both. One difficulty is that no strategy-proof mechanism exists that is both efficient and fair under the presence of minimum quotas. Furthermore, existing mechanisms require that all students consider all schools acceptable to obtain a feasible matching that respects minimum quotas. This assumption is unrealistic in a school choice program.

We consider the environment where a student considers her initial endowment school acceptable and the initial endowments satisfy all the minimum quotas. We develop two strategy-proof mechanisms. One mechanism, which we call the Top Trading Cycles among Representatives with Supplementary Seats (TTCR-SS), is based on the Top Trading Cycles (TTC) mechanism and is significantly extended to handle the supplementary seats of schools while respecting minimum quotas. TTCR-SS is Pareto efficient. The other mechanism, which we call Priority List-based Deferred Acceptance with Minimum Quotas (PLDA-MQ), is based on the Deferred Acceptance (DA) mechanism. PLDA-MQ is fair, satisfies a concept called Priority List-based (PL-) stability, and obtains the student-optimal matching within all PL-stable matchings. Our simulation results show that our new mechanisms are significantly better than simple extensions of the existing mechanisms.

☆ This paper is partially based on the authors' conference publication [30], where TTCR-SS was presented. In this paper, we introduce another mechanism (Section 4) and compare two mechanisms by simulations (Section 5). We also add new theoretical results of TTCR-SS in Section 6.

* Corresponding author.
E-mail addresses: nhamada@agent.inf.kyushu-u.ac.jp (N. Hamada), chialinghsu1982@gmail.com (C.-L. Hsu), kurata@agent.inf.kyushu-u.ac.jp (R. Kurata), t.suzuki@gifu.shotoku.ac.jp (T. Suzuki), sgrueda@cc.saga-u.ac.jp (S. Ueda), yokoo@inf.kyushu-u.ac.jp (M. Yokoo).

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1. Introduction

Traditionally, a student who plans to attend a public school is assigned to one based on where she lives. School choice programs are implemented to give students and their parents opportunities to choose which public schools to attend. In such programs, students submit their preferences over schools to a centralized matching mechanism, which assigns students to schools. A seminal work by Abdulkadiroğlu and Sönmez [3] introduced the idea of using a mechanism design approach to study this issue by formalizing it as a problem of allocating indivisible objects with multiple supplies (seats in schools) to agents (students). This problem is referred to as the school choice problem.

In this paper, we consider a school choice problem with two requirements. First, we assume that a minimum quota constraint is imposed on each school. A school is required to enroll a certain number of students. This is a reasonable assumption since each school needs a minimum number of students to operate. Second, we assume that each student has a default school that she would have attended without a school choice program, which we refer to as her *initial endowment*. We further assume that initial endowments satisfy all minimum quotas. The objective of this paper is to design school choice mechanisms so that each student who participates in the matching process will be able to attend a school that is at least as good as her initial endowment school. On the other hand, strategy-proofness, i.e., no student ever has any incentive to misreport her preference regardless of other students' reports, is critical to many policymakers. We focus on strategy-proof mechanisms in this paper.

Several desirable properties of a matching mechanism have been proposed in the literature. Two widely discussed properties are Pareto efficiency and stability. Pareto efficiency is a welfare notion that rules out incidents that can improve agents' well-being without making others worse off. Stability rules out *justified envy*, which is an incident that violates priority in a school.[1] However, [3] showed that a matching mechanism cannot be both stable and Pareto efficient in the setting of school choice problem, even when there is no distributional constraint. As a result, a policymaker needs to choose between Pareto efficiency and stability.

In this paper, we develop two strategy-proof mechanisms: the *Top Trading Cycles among Representatives with Supplementary Seats* (TTCR-SS) and the *Priority List-based Deferred Acceptance with Minimum Quotas* (PLDA-MQ). The first achieves Pareto efficiency,[2] and the second achieves PL-stability, which is a version of stability we consider in this paper.

Before we introduce our mechanisms, we first introduce two simple mechanisms that can handle minimum quotas. The first is based on the *Top Trading Cycles* (TTC) mechanism of [44]. Since in our setting, each student is endowed with a seat in a default school, a simple way to improve students' welfare is to allow them to trade their seats in schools. Moreover, since we assume that the initial matching satisfies maximum and minimum quotas, the new matching that resulted from trading also satisfies these distributional constraints. This is because trading happens only when a group of students wants to exchange seats, and therefore, the numbers of students who are matched to a school are the same both in the initial and new matching. We call this simple mechanism the *Top Trading Cycles among Representatives* (TTCR). Note that we design a list over the students, the *Master List* (ML), in TTCR to prioritize their rights to form a trading cycle.[3] Note that a student will exchange her seat in her default school with another student only when she can obtain a seat in a school that she prefers to her default school. Thus, in the new matching, each student is weakly better off.

The second simple mechanism is the *Artificial Cap Deferred Acceptance* (ACDA), which is identical as the *Deferred Acceptance* (DA) mechanism [16] except for two adjustments. First, we created an artificial maximum quota for each school that is equal to the number of students who are matched to this school in the initial matching. The mechanism uses artificial maximum quotas instead of true maximum quotas to create matchings. Second, we adjust the priorities of the students in schools so that a student has higher priority in her default school than a student whose default school is different. These two adjustments guarantee that the new matchings created by DA satisfy the distributional constraints and that every student is weakly better off in the new matching.

ACDA is a popular mechanism to handle minimum quotas in practice,[4] and its properties have been analyzed in several studies, for example, [15] and [23]. The real-world applications of ACDA include the hospital-doctor matching in Japan and the cadet-branch matching in the United States ([15] and [23]).

We find that both TTCR and ACDA have severe shortcomings emanating from the fact that the number of students who are assigned to a school is weakly less than its capacity in both mechanisms. The mechanisms developed in this paper, TTCR-SS and PLDA-MQ, are designed to properly exploit these extra seats. TTCR-SS is designed to achieve efficiency, and PLDA-MQ is designed to improve students' welfare while achieving a certain degree of fairness.

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[1] Individual rationality, fairness, and nonwastefulness constitute stability [5]. In our setting, a matching is fair if any school, which a student prefers to her matched school, is occupied by students with higher priority or who initially endow the school. A matching is nonwasteful if moving a student from her currently assigned school to a more preferred one violates minimum or maximum quotas.

[2] In our setting, a Pareto efficient matching is not Pareto dominated by another feasible matching. A feasible matching satisfies both maximum and minimum quotas.

[3] In real-world applications, their GPAs can be used to create ML.

[4] To the best of our knowledge, in practice, ACDA is used in settings where there are no initial endowments. The second adjustment mentioned above is not used in such a setting. We introduce the second adjustment to guarantee that the matching created by this mechanism makes every student weakly better off in the new matching. In the following discussion in this paper, we continue to use the second adjustment.