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Arbitrary arrow update logic ☆

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ABSTRACT

In this paper we introduce arbitrary arrow update logic (AAUL). The logic AAUL takes arrow update logic, a dynamic epistemic logic where the accessibility relations of agents are updated rather than the set of possible worlds, and adds a quantifier over such arrow updates.

We investigate the relative expressivity of AAUL compared to other logics, most notably arbitrary public announcement logic (APAL). Additionally, we show that the model checking problem for AAUL is PSPACE-complete. Finally, we introduce a proof system for AAUL, and prove it to be sound and complete.

1. Introduction

In dynamic epistemic logic [16] various information changing events can be modeled, from modest public announcements, to powerful action models that can change an epistemic model beyond recognition. Here, we study arrow updates, a type of information changing event that is more powerful than public announcements but less powerful than action models. Roughly speaking, in public announcement logic (PAL [28]) one specifies which *states* in the model will remain as a result of the announcement, in arrow update logic (AUL [22]) one puts constraints on pairs in *relations* that endure the update (while in action model logic, also new states and new pairs can emerge as a result of the action). Let us emphasize at this point that although such relations can denote indistinguishability for an agent between states, they can also denote any kind of transition between states, or a temporal relation, a preference, etc. In other words, arrow update logic is relevant for many logics that are used in Artificial Intelligence, whether these logics model epistemic, doxastic or other attitudes of agents, dynamics, strategic interaction, or systems of norms (see also Section 2).

One line of dynamic epistemic logics adds quantifiers over information changing events, ranging from quantifiers over public announcements [10,5], group announcements [2], to quantifiers over action models [5]. An overview of the literature on this topic is provided by [14]. These different "quantified operator" logics find their application in analyzing the concept of knowability [10], but also in, e.g., security where one can express properties like no information changing event can

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disclose certain information to some agent. Such logics with quantifiers over information change find an application in epistemic protocol logics [32,15] that allow for protocol change or protocol declaration. For a different approach to quantification over information change see [8], where a first-order modal logic is used.

In this paper we introduce *arbitrary arrow update logic* (AAUL), which allows quantification over arrow updates. Like the other quantified logics, we can use AAUL to reason about knowability and security. Additionally, AAUL can be used to reason about protocol and rule design, as we will show in Section 2.

We establish three kinds of results concerning AAUL. The first concerns expressivity of the logic. We show that, under the usual assumptions that the set of agents is finite and the set of propositional variables is infinite, arbitrary public announcement logic and arbitrary arrow update logic are incomparable in expressivity over the class of all Kripke models. We also identify a case where AAUL is more expressive than APAL. Finally, we compare arbitrary arrow update logic to a number of other logics, and conclude that it is incomparable to epistemic logic with common knowledge and that it is more expressive than basic epistemic logic (and therefore also more expressive than arbitrary action model logic and refinement modal logic [12]). Secondly, we show that the model checking problem for AAUL is PSPACE-complete. Finally, we introduce a proof system for AAUL, and prove it to be sound and complete with respect to our set of intended models.

To argue for the relevance of AAUL for Artificial Intelligence in general and knowledge representation in particular, it is helpful to also show why AUL is relevant, and to keep in mind that AAUL is to AUL what APAL is to PAL. Where in PAL, semantically (that is, on Kripke models), the object of study is the elimination of states that do not satisfy a given specification (the announcement), in APAL then the question is what kind of sets can be eliminated, and which properties are invariant under arbitrary elimination. As pointed out above, PAL and APAL are primarily studied in contexts where the states represent epistemic information of agents, so PAL and APAL are pre-dominantly used as formalisms to study dynamic epistemic phenomena, answering questions like what kind of information can be learned ('for which φ is $[\varphi]\Box_a\varphi$ true?'), and what kind of information is knowable ('for which φ is there an announcement ψ such that $[\psi]\Box_a\varphi$?'). But elimination of states is also relevant in other contexts then epistemic ones, like for instance in deontic reasoning, where some states may be (morally, or deontically) better than other states. In this context, the PAL construct $[\psi]\varphi$ would be interpreted as 'if a law guaranteeing ψ would be enforced, as a result, φ would be true'.

Where PAL and APAL focus on the elimination of specified or arbitrary sets of states, respectively, the focus of attention of AUL and AAUL is on the elimination of specified or arbitrary sets of *transitions*. For instance, where the deontic interpretation of (A)PAL addresses ought-to-be norms (*'Sein Sollen'*), a deontic interpretation of (A)AUL is about ought-to-do norms (*'Tun Sollen'*), see e.g., the chapter 'Deontic logic as I see it', by the founder of deontic logic, von Wright, in [26] or [13] for a computer science perspective. So if the relations in the Kripke model represent transitions, AUL can be used to reason about social laws: is it the case that, by disallowing certain transitions, we can guarantee a particular property? Norms can relate to rationality for instance, and indeed, in AUL one can mimic backward induction in an extensive form game by requiring that all moves for agent i should be kept which do not affect his chances of winning the game. But then, under this perspective, AAUL is useful for the *Syntheses* problem in social laws, and the *mechanism design* problem in game theory, because it allows one to study questions like 'is there a social law (in the sense that only certain transitions are allowed) that guarantees a certain outcome?' Or, 'is there a game (in the sense that only certain moves in the extensive form of it are allowed) that only leaves a specified set of outcomes?'. The application of AUL and AAUL to social laws and mechanism design in further studied in [25,23]. We return to the normative interpretation of arrow updates in Section 2.2.

Arrow updates also have epistemic interpretations, which reinforces their relevance for knowledge representation. As we will argue in Section 2.1, arrow updates are more general than public announcements, since one can model *semi-private* announcements. These are announcements where only a sub-group of all the agents learn certain information, while all agents are aware what the protocol is (like when all students in a class know that their teacher has sent their marks to the administration office). This implies that AAUL provides a formalism to reason about arbitrary semi-private announcements, making it possible to express properties that are relevant for epistemic planning, like 'there is a private announcement, such that everybody in Ag_1 knows what the password to the system is, while everybody in Ag_2 remains ignorant about this password'. The application of (A)AUL to doxastic logic would have a similar taste as that to epistemic logic. To give a simple example, removing a reflexive arrow in doxastic logic would correspond to a situation where an agent's belief are not necessary correct any more.

More generally, in every AI-context where Kripke models are used to represent information in a certain context, AUL and AAUL can be applied to reason about a dynamic representation of that context, where certain transitions between certain states can be removed. If the binary relations represent agents who can take moves, AUL enables us to reason about forbidding certain agents to act in certain situations, wheres AAUL can represent information about what can be achieved in principle, by restricting the moves that are available to the agents. If the accessibility relation represents the flow of time, AAUL can formulate questions of what is guaranteed to hold if certain transitions will not occur. The relation in a Kripke model could represent what the goals are of agents: AAUL in this case would provide a formalism to reason about agents dropping goals, which is considered to be an important aspect in agent programming languages (see for instance the programming language GOAL [21,1]). Likewise AUL and AAUL provide tools to reason about intention revision [31] and hence, in principle for the dynamics of many agents' attitudes, including Beliefs, Desires and Intentions [29].

The arbitrary arrow update operator in AAUL adds *implicit* quantification over arrow updates. Recently, [9] used the capacity in second order modal logic to *explicitly* quantify over propositions. This makes it possible to define arbitrary announcements within the object language: $\forall p[p]\varphi$. Additionally, this also makes it possible to express properties like