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Structural reliability analysis with fuzzy random variables using error principle

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ABSTRACT

In structural reliability calculation, there are fuzzy uncertainties in the distribution parameters of random variables, which bring the problem of large computation and poor precision. In order to improve the accuracy and efficiency of structural reliability, a novel structural reliability calculation method with fuzzy random variables is proposed from the perspective of error propagation. Firstly, fuzzy variables are transformed into uncertain interval variables according to the fuzzy decomposition theorem. Secondly, by using the error transfer principle, the $sigk\,(.)$ function is introduced into the reliability function to approximate the step function, and a structural reliability error analysis model based on the direct integration method is established. On this basis, the equivalent error of the fuzzy variable is determined by traversing the interval value of the membership function at [0, 1] level cut set, and then the structural reliability interval values corresponding to each cut set are obtained. The examples are investigated to demonstrate the efficiency and accuracy of the proposed method, which provides a feasible way to analyze and calculate the structural reliability with uncertain variables such as fuzzy random variables, random variables and fuzzy variables.

1. Introduction

Structural reliability is proposed due to a large number of uncertainties in structural engineering. With people's deepening research on structural reliability, it is found that there exists not only random, but also widely fuzzy in many practical structural engineering problems. Many fuzzy uncertainties exist in the process of structural analysis design, such as the failure wear size of mechanical parts and the maximum deformation displacement, etc.

In the structural reliability analysis methods, the development of probabilistic reliability and non-probabilistic reliability method are relatively mature. The probabilistic reliability analysis methods with random variables include moments method (Kato et al., 2002; Zhao and Ono, 1999, 2000) (such as first-order second moment, second-order second moment, high order moment method, etc.), response surface method (Li et al., 2011, 2015), Monte Carlo method (Coppens et al., 2006), direct integration method (Zhang et al., 2011), etc. The non-probabilistic reliability analysis methods with interval variables include convex model method (Jiang et al., 2014), possibility theory method (Li et al., 2016), interval analysis method (Guo et al., 2001), evidence theory method (Jiang et al., 2013), etc.

At present, for fuzzy reliability analysis fuzzy variables are often converted into random variables or interval variables and fuzzy reliability problems are solved by probabilistic reliability or non-probabilistic reliability method. In Zhao et al. (2010) random and fuzzy variables were converted into each other equivalently, and then traditional reliability theory was used to analyze fuzzy reliability problems. In Song and Lu (2008) the integral area of general failure probability was discretized according to the performance functions. In the integral area of discretization, the membership function of the limit state function attaching to fuzzy failure domain almost remained constant. Fuzzy reliability problems were transformed into general reliability problems and general failure probability was solved with approximate moment method. Guo et al. (2002) used fuzzy variables to describe structural uncertainty. The reliability model for fuzzy structure and system was established based on the possibility theory and the fuzzy interval analysis. In Song and Qiu (2013) the fuzzy non-probabilistic reliability model with fuzzy-interval mixed variables was established by converting fuzzy variables into interval variables.

At the same time, the concept of fuzzy random variable (Kwakernaak, 1978; Puri and Dan, 2001) is proposed, and the fuzziness and randomness of structural uncertainty are organically combined

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about the probability density distribution parameters. A fuzzy random variable is a random variable for which the parameters of its distribution (e.g., mean and standard deviation) are considered fuzzy numbers (Jahani et al., 2014). Random variables, fuzzy variables can be regarded as special cases of fuzzy random variables. Fuzzy random variables have more applicability. Hai et al. (2009) proposed a fuzzy reliability analysis method with equivalent probability density function according to basic concepts of randomness and fuzziness. Based on the membership function of fuzzy distribution parameters, the prior distribution of random probability density function was constructed, the equivalent probability density function with fuzzy distribution parameters was obtained, and some common membership functions were derived, which was suitable for reliability calculation with multiple fuzzy distribution parameters. In Zao and Qiu (2009), the structural reliability calculation method of fuzzy random parameters of the stress and strength was given according to fuzzy decomposition theorem, interval extension principle and a second order moment method. In Tan et al. (2009) the calculation method of reliability index of slope stability was proposed on basis of fuzzy decomposition theorem, interval extension principle and design point method. In Xu and Qiu (2014) the calculation method of structural reliability was proposed according to the fuzzy decomposition theorem and the hybrid variable non-probabilistic reliability analysis algorithm. The simulation results of structural reliability under different level cut were obtained. In Wang et al. (2012), aiming at the reliability problem of fuzzy random variable parameter, the structural performance function and its former fourth order cumulates were calculated by the random sampling of fuzzy variables, and the interval value of structural reliability was estimated by the saddle point approximation principle. In Cui et al. (2011) according to the structural reliability of fuzzy or interval variables of random distribution parameters, the structure failure probability was expressed as product form of distributed parameter probability and structure failure probability with distributed parameter. The structural reliability calculation method of fuzzy random variables was obtained. In fact, it was an application of fuzzy decomposition theorem, Bayes theory and Monte method in reliability calculation. In Jahani et al. (2014) reliability evaluation of the structure with uncertain load and material characteristics has been done. Using the fuzzy random variable model, the failure probability evaluation was made with interval Monte Carlo method and interval finite element method.

At present, for the reliability problems with fuzzy random variables there are a lot of algorithms, such as the transformation into interval variables and the Bayesian theory method. But they have many problems. When it is converted into interval variable, there are large amount of calculation and existence of interval expansion, etc. The method based on Bayes theory is complex, not easy to be accepted by engineering technology, and has disadvantages of poor practicability. In view of the disadvantages of the above methods, this paper proposes a novel method about fuzzy reliability calculation with fuzzy random variables. The error analysis and error transfer theory are applied to fuzzy reliability according to the conceptual similarity of uncertainty interval variable and undetermined system error. Based on fuzzy decomposition theorem, the fuzzy variables are transformed into a series of interval variables under different level cut set. The interval variable is equivalent to the random variable of uniform distribution, and its mean and variance are calculated. The structural reliability function of fuzzy variable is established, and the reliability mean is obtained in the mean of equivalent uniform random variable. Then, the reliability error transfer function is derived from the whole differential relationship of function. The standard deviation of the equivalent random variable is considered as error to calculate reliability error range. By superimposing the reliability mean and the error, the reliability interval value is obtained, and the fuzzy reliability is obtained by the fuzzy decomposition theorem. Compared with the traditional method of converting fuzzy variables into interval variables, this method improves the computational efficiency of structural fuzzy reliability under the premise of ensuring the accuracy of calculation.

This paper is organized as follows. In Section 2, equivalent error of fuzzy parameters is calculated. In Section 3, error analysis method of structural reliability with fuzzy random variables is proposed. Examples are followed to demonstrate the proposed methods in Section 4. Conclusions are arrived in Section 5.

2. Equivalent error of fuzzy parameters and error transfer in function

2.1. Calculation of equivalent error of fuzzy parameters

According to fuzzy decomposition principle, any fuzzy variables can be decomposed into a series of interval variables under different level cut set.

Define A as a fuzzy subset on the universe X, A_λ is the λ-cut set of A, $\lambda \in [0,1]$, so the segmentation type is as follows:

$$\widetilde{A} = \bigcup_{\lambda \in [0,1]} \lambda A_\lambda \tag{1}$$

Where, $A_\lambda = \left[a_\lambda b_\lambda\right]$.

Then the interval variables A_λ under each level cut λ are equivalent to the random variable of uniform distribution. The mean of the equivalent random variable is $\mu_A^\lambda = \left(a_\lambda + b_\lambda\right)/2$ and the standard deviation is $\sigma_A^\lambda = \sqrt{\left(b_\lambda - a_\lambda\right)^2/12}$.

The standard deviation σ_A^λ of the equivalent random variable is regarded as the error δ_A^λ, and the equivalent error of fuzzy parameter $\widetilde{A}$ under level cut set λ can be obtained $\delta_A^\lambda = \sigma_A^\lambda$.

2.2. Error transfer principle

According to the transfer of error function, if the error of variables is known, the indirect error of function can be seen.

Assume that the function is $y = f\left(x_1, x_2, \dots, x_n\right), \delta x_1, \delta x_2, \dots, \delta x_n$ are respectively the errors of $x_1, x_2, \dots, x_n$ and $X_1, X_2, \dots, X_n$ are the true value of $x_1, x_2, \dots, x_n$. Hence, the error of y is as follows:

$$\delta y = f\left(x_1, x_2, \dots, x_n\right) - f\left(X_1, X_2, \dots, X_n\right) \tag{2}$$

The Taylor series expansion of $y = f\left(x_1, x_2, \dots, x_n\right)$ is given and ignore 2nd order Taylor series expansion. $y = f\left(x_1, x_2, \dots, x_n\right)$ is transformed as follows:

$$\begin{aligned} y &= f\left(x_1, x_2, \dots, x_n\right) \\ &= f\left(X_1, X_2, \dots, X_n\right) + \left(\frac{\partial f}{\partial x_1}\right)\left(x_1 - X_1\right) + \left(\frac{\partial f}{\partial x_2}\right)\left(x_2 - X_2\right) \\ &\quad + \cdots + \left(\frac{\partial f}{\partial x_n}\right)\left(x_n - X_n\right) \\ &= f\left(X_1, X_2, \dots, X_n\right) + \left(\frac{\partial f}{\partial x_1}\right)\delta x_1 + \left(\frac{\partial f}{\partial x_2}\right)\delta x_2 + \cdots + \left(\frac{\partial f}{\partial x_n}\right)\delta x_n \end{aligned} \tag{3}$$

Where $\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}$ are respectively partial derivative values of $f\left(x_1, x_2, \dots, x_n\right)$ to $x_1, x_2, \dots, x_n$ in $x_1 = X_1, x_2 = X_2, \dots, x_n = X_n$.

Substituting Eq. (3) into Eq. (2), the function error is as follows:

$$\begin{aligned} \delta y &= f\left(x_1, x_2, \dots, x_n\right) - f\left(X_1, X_2, \dots, X_n\right) \\ &= \left(\frac{\partial f}{\partial x_1}\right)\delta x_1 + \left(\frac{\partial f}{\partial x_2}\right)\delta x_2 + \cdots + \left(\frac{\partial f}{\partial x_n}\right)\delta x_n \\ &= \sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)\delta x_i \end{aligned} \tag{4}$$

The error transfer in the function is realized through the Eq. (4).

3. Error analysis method of structural reliability with fuzzy random variables

3.1. Error analysis model of structural reliability

In structural reliability theory, the probability of failure is $P_f = \int_{g(x)\le 0} f(x)\mathrm{d}x$ (Rackwitz, 2001). Where $f(x)$ denotes the joint probability density function, $g(x)$ denotes the performance function. Hence,