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A morphological neural network for binary classification problems

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ABSTRACT

The dilation–erosion perceptron (DEP) is a hybrid morphological processing unit, composed of a balanced combination between dilation and erosion morphological operators, recently presented in the literature to solve some problems. However, a drawback arises from such model for building complex decision surfaces for non-linearly separable data. In this sense, to overcome this drawback, we present a particular class of morphological neural networks with multilayer structure, called the dilation–erosion neural network (DENN), to deal with binary classification problems. Each processing unit of the DENN is composed by a DEP processing unit. Also, a descending gradient-based learning process is presented to train the DENN, according to ideas from Pessoa and Maragos. Furthermore, we conduct an experimental analysis with the DENN using a relevant set of binary classification problems, and the obtained results indicate similar or superior classification performance to those achieved by classical and state of the art models presented in the literature.

1. Introduction

Morphological neural networks (MNN) (Davidson and Ritter, 1990; Davidson, 1991; Davidson and Talukder, 1993; Ritter and Sussner, 1996) are a particular class of artificial neural networks (ANN) based on fundamental concepts of mathematical morphology (Matheron, 1975; Serra, 1982). The development of MNN has grown due to an important observation made by Ritter et al. (1989, 1990) and Ritter and Wilson (2001), demonstrating that the image algebra (Sternberg, 1985) can be used to develop several ANN models.

In this sense, several MNN have been presented in the literature to solve relevant problems, such as automatic target detection (Khabou and Gader, 2000; Khabou et al., 2000), landmine detection (Gader et al., 2000), rule extraction (Kaburlasos and Petridis, 2000), handwritten character recognition (Pessoa and Maragos, 2000), image restoration (Sousa et al., 2000), hyperspectral image analysis (Graña et al., 2003; Ritter et al., 2009), time series forecasting (Sussner and Valle, 2007; Araújo, 2011, 2013), pattern recognition (Sussner and Esmi, 2011a; Jamshidi and Kaburlasos, 2014), software development cost estimation (Araújo et al., 2012; Araújo, 2012), computer vision (Sussner et al., 2012), vision-based self-localization in mobile robotics (Esmi et al., 2014), high-frequency financial data prediction (Araújo et al., 2015), image understanding (na and Chyzhyk, 2016), amongst others.

In general, an MNN performs morphological operators among complete lattices within each processing unit of the network. The reason for that is because the complete lattice theory is accepted as theoretical framework for the mathematical morphology (Serra, 1988; Ronse, 1990; Banon and Barrera, 1993; Heijmans, 1994). Therefore, the set of all structuring elements of morphological operators, employed within each processing unit of an MNN, represents the network's weights (Sussner and Esmi, 2011a). In the context of MNNs, it is worth mentioning the existence of a hybrid morphological processing unit, called the dilation–erosion perceptron (DEP) (Araújo, 2011, 2013), which was recently presented in the literature to solve some nonlinear problems. Such a model can be represented by a balanced combination between morphological operators (dilation and erosion) within an unique operator with hybrid morphological structure.

However, Araújo (2011, 2013) argue that only one processing unit is not able to produce complex decision surfaces to classify efficiently non-linearly separable data, and this is the main drawback of the DEP model. In this sense, as future work, Araújo (2011, 2013) have suggested the development of a neural network with multilayer structure using the DEP as fundamental processing unit to overcome these limitations.

In this sense, in order to overcome such drawback, we propose to extend the main ideas presented in Araújo (2011, 2013) to a neural network with a multilayer feed-forward structure, called the dilation–erosion neural network (DENN). Each morphological processing unit of the DENN model is represented by a balanced combination between dilation and erosion operators, followed by an activation function.

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Besides, based on Pessoa and Maragos (2000), a gradient-based learning process is presented to train the DENN model, where a smoothed rank-based approach is employed to overcome the nondifferentiability problem of dilation and erosion morphological operators. At the end, we conduct an experimental analysis with the proposed DENN using a relevant set of binary classification problems, where three measures are used to assess performance and compare to classical and state of the art models presented in the literature.

Therefore, the main contributions of this work are: (i) proposal of an architecture of morphological neural network to deal with binary classification problems, (ii) proposal of a descending gradient-based approach using ideas from the back-propagation algorithm to train the proposed model, and (iii) Performing an experimental analysis with benchmark datasets to assess classification performance.

This work is organized as follows. Section 2 presents the needed concepts about lattice theory for morphological neural networks. Section 3 presents the formal definition of the proposed model. Section 4 presents the descending gradient-based learning process to train the proposed model. Section 5 presents the simulations and experimental results. At the end, in Section 6, we present the final remarks of this work.

2. Background on morphological neural networks

The mathematical morphology (MM) (Matheron, 1975; Serra, 1982) represents a theory initially applied for image processing and image analysis using structuring elements (SE) (Serra, 1982, 1988; Heijmans, 1994; Soille, 1999). In this context, Ritter et al. (1989, 1990) and Ritter and Wilson (2001) have demonstrated that the image algebra (Sternberg, 1985) can be used to develop a particular class of artificial neural networks, called the morphological neural network (MNN) (Davidson and Ritter, 1990; Davidson, 1991; Davidson and Talukder, 1993; Ritter and Sussner, 1996).

In this sense, we can observe that MNN are based on the complete lattice theory (Ritter et al., 1997; Ritter and Sussner, 1997; Petridis and Kaburlasos, 1998; Kaburlasos and Petridis, 2000; Ritter and Urcid, 2003; Ritter et al., 2004; Sussner and Esmi, 2011b), since the lattice theory (Dedekind, 1987; Birkhoff, 1993) can be seen as a theoretical-framework of the MM (Serra, 1988; Ronse, 1990; Banon and Barrera, 1993; Heijmans, 1994). Next, we present some fundamental concepts and relevant theories for the development of the DENN. Additional information can be found in Banon and Barrera (1993), Birkhoff (1993), Cuninghame-Green (1995), Ritter and Urcid (2003), Sussner and Valle (2006), Araújo and Susner (2010) and Sussner and Esmi (2011a).

Definition 2.1. Let X be a nonempty set, and let $\leq$ be a binary relation in X, then $\leq$ is a partial order if and only if ($\forall\ x,\ y,\ z \in A$):

(1) $x \leq x$;
(2) $x \leq y$ and $y \leq x \Rightarrow x = y$;
(3) $x \leq y$ and $y \leq z \Rightarrow x \leq z$.

Definition 2.2. Let $(X, \leq)$ be a partially ordered set and let Y a subset of X ($Y \subseteq X$). Then $x \in X$ is an upper bound of Y if and only if $y \leq x$, $\forall\ y \in Y$. Similarly, $x \in X$ is a lower bound of Y if and only if $x \leq y$, $\forall$ $y \in B$.

Note that the smallest upper bound of Y is defined as the supremum of Y and denoted by $\bigvee_{i\in I} y^i$ when $Y = \{y^i, i \in I\}$ for a finite index set I. Similarly, the greatest lower bound of Y is defined as the infimum of Y and denoted by $\bigwedge_{i\in I} y^i$ when $Y = \{y^i, i \in I\}$.

Definition 2.3. A partially ordered set L is a lattice if and only if all finite subset of L has a supremum and an infimum in L, that is, $\forall\ X \subseteq L$ finite then $\bigvee X \in L$ and $\bigwedge X \in L$.

Note that a lattice L is a bounded if and only if it has at least element 0_L and a greatest element 1_L.

Definition 2.4. A lattice L is a complete lattice if and only if all finite or infinite subset of L has a supremum and an infimum in L, that is, $\forall$ $X \subseteq L$ finite or infinite we have $\bigvee X \in L$ and $\bigwedge X \in L$.

Definition 2.5. Let L and M be complete lattices, let δ and ε operators from the complete lattice L to the complete lattice M, and let $X \subseteq L$. Then, we have

(1) The operator δ is an algebraic dilation if and only if

$$\delta\left(\bigvee X\right) = \bigvee_{x\in X} \delta(x). \tag{1}$$

(2) The operator ε is an algebraic erosion if and only if

$$\varepsilon\left(\bigwedge X\right) = \bigwedge_{x\in X} \varepsilon(x). \tag{2}$$

Banon and Barrera (1993) have presented important theorems regarding the decomposition of mappings among complete lattices in terms of elementary morphological operators. In this work we focus on the decomposition of increasing operators, since morphological operators of dilation and erosion are increasing.

Definition 2.6. Let L and M be lattices. An operator $\Psi : L \to M$ is increasing if and only if $(\forall x, y \in L)$:

$$x \leq y \Rightarrow \Psi(x) \leq \Psi(y). \tag{3}$$

In this way, the Banon and Barrera decomposition for increasing operators leads to the theorem presented as follows.

Theorem 2.1. *Let L and M be complete lattices, and let $\Psi : L \to M$ be an increasing mapping, then we have*

(1) *There are dilations δ^i for an index set I so that*

$$\Psi = \bigwedge_{i\in I} \delta^i. \tag{4}$$

(2) *There are erosions ε^i for an index set I so that*

$$\Psi = \bigvee_{i\in I} \varepsilon^i. \tag{5}$$

It is worth mentioning that dilation and erosion operators require an additional algebraic structure besides the complete lattice structure. In this way, we use the same extension proposed by Green (1979) and Cuninghame-Green (1995) and employed by Sussner and Esmi (2011a). Some fundamental concepts os this approach is presented as follows.

Definition 2.7. Let G be a nonempty set, and let $\leq$ a partial order relation. Then G is a group if and only if

(1) G is a lattice;
(2) $\forall a, b, x, y \in G$, then $a \leq b \Rightarrow xay \leq xby$.

Definition 2.8. Let G be a lattice and also a group, and let "$+$" an addition operation among groups. Then, the group G is a lattice ordered group if every group translation using the operation "$+$" be isotone, that is, if and only if ($\forall\ x,\ y \in G$ so that $x \leq y$):

$$a + x + b \leq a + y + b, \quad \forall a, b \in G. \tag{6}$$

Let "$+'$" an addition operation over $(G\times G)\setminus\{(+\infty, -\infty), (-\infty, +\infty)\}$, then "$+'$" differs from conventional addition operation "$+$" according to the following rules:

$$(-\infty) + (+\infty) = (+\infty) + (-\infty) = -\infty, \tag{7}$$

and

$$(-\infty) +' (+\infty) = (+\infty) +' (-\infty) = +\infty. \tag{8}$$