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Ordered weighted aggregation of fuzzy similarity relations and its application to detecting water treatment plant malfunction

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ABSTRACT

Ordered weighted aggregation procedures have been introduced in many applications with promising results. In this paper, an innovative approach for ordered weighted aggregation of fuzzy relations is proposed. It allows the integration of component relations generated from different perspectives of a certain observation to form an overall fuzzy relation, deriving a useful similarity measure for observed data points. Two types of aggregation are investigated: (a) min/max operators are employed for the aggregation of component relations defined by the minimum T-norm; and (b) sum/product operators are employed for the aggregation of component relations defined by the Łukasiewicz T-norm. The resultant ordered weighted aggregations prove to preserve the desirable reflexivity and symmetry properties, with T-transitivity also conditionally preserved if appropriate weighting vectors are adopted. The conditions upon which the proposed aggregated relations preserve T-transitivity are studied. The characteristics of applying an aggregated relation in combination with clustering procedures is also experimentally examined, where fuzzy similarity relations regarding individual features are aggregated to support hierarchical clustering. An application to the detection of water treatment plant malfunction demonstrates that better results can be obtained with the transitive fuzzy relations acting as the required similarity measures, as compared to the use of non-transitive ones. By introducing transitivity to the aggregation the interpretability of the detection system is also enriched.

1. Introduction

Methods for aggregation of different pieces of information into an integrated form are an indispensable tool, not only for theoretical development in e.g., mathematics and physics, but for many real-world applications in engineering, economical, social, and other fields. Having recognised this, a significant number of aggregation operators have been developed, ranging from simple arithmetic mean to more complicated fuzzy methods, including minimum/maximum, uninorm, and other alternative T-norms/T-conorms (Calvo et al., 2012; Beliakov et al., 2008; Calvo et al., 2002). In particular, a class of parameterised mean-like aggregation operators, commonly named as ordered weighted averaging (OWA), have been introduced in the literature (Yager, 1988) and successfully applied in different areas (Chen and Zhou, 2011; Merigó and Casanovas, 2011; Suo et al., 2012; Su et al., 2016). Intuitively, with an appropriate specification of a weighting vector, an OWA operator helps to capture and reflect the uncertain nature of human judgements in problem-solving, generating an aggregated result that lies between the (conventional) two extremes of minimum or maximum combination of multi-featured data objects (Yager, 2010).

In general, relations holding amongst data points form the basis for many developments and applications of fuzzy systems. However, in their applications to supporting multicriteria decision making (Li et al., 2015; Williams and Steele, 2002), which forms a major challenge for practical fuzzy systems, a key question is what underlying properties of the data can be preserved in the process of constructing or aggregating similarity relations. For certain applications like prototype-based reasoning where clusters of objects that are similar to certain prototypical samples are sought (Perner, 2002), properties such as reflexivity and transitivity (Tolias et al., 2001) may not be necessary. Yet, there are many other situations in which it is desirable to maintain the symmetry and a degree of transitivity over the homogeneous similarity classes or granules whose members possess these properties as symmetric and transitive classes or granules support intuitive interpretation of the reasoning process involved (Fernández Salido and Murakami, 2003; Lifen, 2008; Wittkop et al., 2010).

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To enhance the mechanism for aggregation of fuzzy relations with such desired properties entailed, this paper presents two novel types of OWA-based aggregation methods, where the component relations are sorted first and subsequently aggregated with assigned weights. These techniques allow the aggregated results to retain the $T_{\min}$-transitive and $T_Ł$-transitive similarities, respectively. It is theoretically proven that the aggregated relations can hold the respective T-transitivity if the weights are arranged in ascending order. To illustrate the effectiveness of such ordered weighted aggregation of fuzzy relations, it is systematically evaluated over the task of clustering both synthetical and UCI datasets, by following the strategy of hierarchical clustering. In this experimental evaluation, similarities between data patterns are measured through ordered weighted aggregation of component fuzzy relations which hold amongst individual features. The work is applied to the detection of water treatment plant malfunction, demonstrating that the aggregated $T_Ł$-transitive similarities lead to better hierarchical clusters than those of non-transitive similarities.

The paper is organised as follows. Section 2 introduces the basic concepts of the aggregation of fuzzy relations. Section 3 presents two types of ordered weighted aggregation of fuzzy relations, with a detailed discussion of their properties, including the use of stress functions to decide on the weighting vectors for them. Section 4 describes the experimental investigation into the proposed aggregation of fuzzy relations in performing clustering tasks, evaluated over a number of classic datasets. Section 5 presents an application of the proposed aggregator to detecting malfunctions of a water treatment plant. The paper is concluded in Section 6, with a discussion of further research.

2. Preliminaries

2.1. Fuzzy relations

The concept of similarity is a preliminary notion in human cognition, playing an essential role in many tasks such as taxonomy, recognition, and inference (e.g., case-based reasoning). Particularly, fuzzy sets and relations (Zadeh, 1971) are of great significance in both theoretical development and industrial applications of constructing similarity metrics when dealing with imprecise situations (Savino and Sekhari, 2009; Su et al., 2013; Savino et al., 2017).

Definition 1. Let X be a nonempty universe. A fuzzy relation $R = [r(a,b)] : X \times X \to [0,1]$ is

- reflexive iff $\forall a \in X, r(a,a) = 1$;
- symmetric iff $\forall a, b \in X, r(a,b) = r(b,a)$;
- T-transitive iff $\forall a, b, c \in X, r(a,b) \geq T(r(a,c), r(c,b))$,

where T is a T-norm (Schweizer and Sklar, 2011), e.g., a mapping $T(x,y) : [0,1] \times [0,1] \to [0,1]$ which satisfies

(1) commutativity: $T(x,y) = T(y,x)$;
(2) monotonicity: $T(x,y) \leq T(x',y')$, if $x \leq x'$ and $y \leq y'$;
(3) associativity: $T(x, T(y,z)) = T(T(x,y), z)$; and
(4) boundary condition: $T(x,1) = x$.

A number of T-norms have been proposed in the literature, including (but not limited to):

- the minimum T-norm: $T_{\min}(x,y) = \min(x,y)$,
- the product T-norm: $T_{\mathrm{p}}(x,y) = x \cdot y$, and
- the Łukasiewicz's T-norm: $T_Ł(x,y) = \max(x+y-1, 0)$.

There exist many different definitions of similarity metrics which have been employed with success for different purpose such as clustering, classification, recognition and diagnostics. However, it is very challenging to validate the effectiveness of a similarity metric in real application scenarios. In this paper, the proposed aggregation methods focus on the use of transitive similarity metrics in support of water treatment plant monitoring.

2.2. Aggregation of fuzzy relations

In describing many engineering problems, an entity is commonly represented by a set of features or evaluated by a set of characteristic indicators (Savino and Apolloni, 2007). As such, the evaluation of similarity between two entities is usually based on their feature/indicator-values. When multiple indicators are considered, an aggregator is typically employed to combine multiple similarity values into a single one. For example, the similarity degrees derived from individual water quality indicators can be aggregated using a weighted sum in an effort to construct an overall water quality index for rivers (Liou et al., 2003). In the following, relevant concepts and properties regarding aggregation of fuzzy relations are introduced.

Formally, let X denote a finite set, $R_j = [r_j(a,b)] : X \times X \to [0,1], a, b \in X, j = 1, \ldots, m$ denote m fuzzy relations (named as component relations) on X, and $w_1, \ldots, w_m \in [0,1]$ denote weights, respectively associated with these relations. The aggregation process aims at providing a relation $R = [r(a,b)], a, b \in X$, summarising the component relations $R_1, \ldots, R_m$ in conjunction with the information implied by the weights $w_1, \ldots, w_m$. Here, the aggregated degree $r(a,b) \in [0,1]$ at position $(a,b), a, b \in X$ depends on the local compositions $r_1(a,b), \ldots, r_m(a,b)$. The component relations usually represent the similarities of patterns from different perspectives such as opinions from different experts, multiple criteria of evaluation and different features of describing data.

Definition 2 (*Fonck et al., 1998*). The aggregation of component relations $R_1 = [r_1(a,b)], \ldots, R_m = [r_m(a,b)], a, b \in X$, with weights $w_1, \ldots, w_m$, is a relation R over X such that

$$r(a,b) = \mathrm{Agg}(r_1(a,b), \ldots, r_m(a,b), w_1, \ldots, w_m) \tag{1}$$

where $a, b \in X$ and Agg is a mapping $[0,1]^{2m} \to [0,1]$, non-decreasing in the first m places and satisfying:

$$\mathrm{Agg}(0, \ldots, 0, w_1, \ldots, w_m) = 0, \text{and } \mathrm{Agg}(1, \ldots, 1, w_1, \ldots, w_m) = 1$$

Both the weighted and non-weighted aggregation procedures have been studied in the literature. For the purpose of aggregating fuzzy relations, typical methods investigated include the norm-conorm and sum–product operators. Usually, the T-norm/conorm operators are employed to aggregate a more general type of fuzzy relations while the sum–product operators are employed to aggregate fuzzy relations which preserve $T_Ł$ transitivity (Fonck et al., 1998; Sudkamp, 1993). An aggregator may be described as optimistic or pessimistic: An optimistic aggregator produces outputs that are closer to the maximum of its inputs, and the outputs of a pessimistic one are closer to the minimum of its inputs.

Definition 3 (*Fonck et al., 1998*). Given component fuzzy relations $R_j = [r_j(a,b)], j = 1, \ldots, m$, the optimistic aggregated fuzzy relation over these relations is

$$R_{opt} = [r_{opt}(a,b)] : \; r_{\mathrm{opt}}(a,b) = S_{j=1,\ldots,m} T(w_j, r_j(a,b)); \tag{2}$$

and the pessimistic aggregated fuzzy relation over these relations is

$$R_{pess} = [r_{pess}(a,b)] : \; r_{\mathrm{pess}}(a,b) = T_{j=1,\ldots,m} S(N(w_j), r_j(a,b)); \tag{3}$$

where, T is a T-norm, S is a T-conorm and N is a strong negation.

Intuitively, the weight w_j here reflects the relative importance of R_j. These two aggregators may be explained with the specific case where all the m weights are assumed to be either 0 or 1 (representing negligible or significant, respectively). In this case, Eq. (2) and Eq. (3) can be rewritten as $R_{\mathrm{opt}} = S_{\{j|w_j=1\}} R_j$ and $R_{\mathrm{pess}} = T_{\{j|w_j=1\}} R_j$, respectively. Thus, $r_{\mathrm{opt}}(a,b)$ can be viewed as the degree of truth of the statement that "there exists at least one significant criterion for which a hold the relation with b", and $r_{\mathrm{pess}}(a,b)$ as the degree of truth of the statement