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Generalized spline nonlinear adaptive filters

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ABSTRACT

A new nonlinear filter, which employs an adaptive spline function as the basis function is designed in this paper. The input signal to this filter is used to generate suitable parameters to update the control points in a spline function. The update rule for updating the control points have been derived and a mean square analysis has been carried out. The output of the spline functions are suitably combined together to obtain the filter response. This filter is called the generalized spline nonlinear adaptive filter (GSNAF). The proposed GSNAF is similar to a functional link artificial neural network (FLANN), considering a functional expansion using spline basis functions. GSNAF has been shown to offer improved accuracy in benchmark classification scenarios and provide enhanced modeling accuracy in single input single output as well as in multiple input multiple output dynamic system identification cases.

1. Introduction

Estimating the mathematical model of a system based on input-output observations is a critical task in many engineering solutions (Ahmad, Azuma & Sugie, 2016; Ewen & Weiner, 1980; Garrido, Curadelli & Ambrosini, 2017; Soderstrom & Stoica, 1988) including satellite communication system (Panicker, Mathews & Sicuranza, 1998), acoustic echo cancellation (Comminiello, Scarpiniti, Azpicueta-Ruiz, Arenas-Garcıa & Uncini, 2014), active noise control (Das & Panda, 2004) and feedback cancellation in digital hearing aids. In a conventional system identification task, an adaptive finite impulse response (FIR) filter is used to model the plant. This is achieved by updating some parameters of the adaptive FIR filter like multiplier values using a suitable algorithm in such a way as to make the model behaves similar to the plant. Such modeling schemes are not effective when nonlinear elements are present in the plant as FIR filters are not designed to handle nonlinearities. Several nonlinear filters have been proposed in the recent past to overcome this limitation of traditional adaptive FIR filter based modeling methods (Gotmare, Patidar & George, 2015).

Popular among such nonlinear modeling approaches are the ones which use an artificial neural network (ANN) as the model. Multi-layer perceptron (MLP) based modeling schemes have been widely reported in literature (Narendra & Parthasarathy, 1990). However, such MLP based system identification methods require update of multiple layers of parameters and thus increase the computational complexity. Radial basis function (RBF) based modeling has also been attempted, which has been shown to provide enhanced modeling accuracy (Shin, 1994). However, such models have been shown to offer higher computational time in the testing phase in comparison with MLP based schemes. Another popular class of nonlinear filters is the ones which are generally called as linear-in-the-parameters nonlinear filters.

The functional link artificial neural network (FLANN) is one of the most commonly used linear-in-the-parameters nonlinear filter (Pao, Phillips & Sobajic, 1992). FLANN is a single layer neural network, the weights of which are usually updated using a gradient descent scheme (Comminiello et al., 2014; Das & Panda, 2004). In a FLANN, the input signal is functionally expanded using a nonlinear function, the expanded signal vector is multiplied by an adaptive weight network and the resulting outputs are combined together to obtain the filter output. The functional expansion employed in a FLANN may be trigonometric, Chebyshev, Legendre or power series (Pao, 1989; Patel, Gandhi, Heda & George, 2016). A dynamic system identification scheme, which uses a trigonometric FLANN has been reported in Patra, Pal, Chatterji and Panda (1999). The authors have shown enhanced modeling accuracy of the FLANN based scheme over MLP based modeling. A Chebyshev FLANN based dynamic system identification has been attempted in Patra and Kot (2002) and a Legendre FLANN based modeling has been presented in Patra and Bornand (2010a).

Sicuranza and Carini (2011) have recently proposed a generalized FLANN (GFLANN), which has introduced cross terms in a trigonometric FLANN. GFLANN has been shown to provide improved noise cancellation over conventional trigonometric FLANN

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in a nonlinear active noise control (ANC) scenario. A set of Fourier nonlinear filters, which offer improved performance were developed in Carini and Sicuranza (2013). One such nonlinear filter is the even mirror Fourier nonlinear (EMFN) filter, which has been reported to provide enhanced accuracy in nonlinear modeling scenarios. EMFN has also been successfully applied for noise cancellation in a nonlinear ANC system (Patel & George, 2015b). It has been also reported that EMFN outperform adaptive Volterra filters, which is a popular class of nonlinear filter, when applied to a nonlinear system identification problem (Carini & Sicuranza, 2014).

A nonlinear adaptive filter, which is a cascade of an adaptive FIR filter and an adaptive spline activation function has been recently introduced in Scarpiniti, Comminiello, Parisi and Uncini (2015). The nonlinear filter is referred to as the spline adaptive filter (SAF), in which, the weights of the adaptive FIR filter as well as the control points or knots of the spline activation function are updated using a gradient descent approach. The most widely used spline functions in signal and image processing areas are *Catmul-Rom* (CR) splines and B-splines (De Boor, 1978). The possibility of designing SAFs based on other popular spline schemes like the ones based in P-spline, L or V curves may explored in the future (Eilers & Marx, 1996; Frasso & Eilers, 2015; Iorio, Frasso, Antonio & Siciliano, 2016). SAF and its variants have found applications in several areas including system identification (Scarpiniti, Comminiello, Parisi & Uncini, 2013) and ANC (Patel & George, 2015a, 2016). In an SAF, the activation function is updated by updating a small set of control points instead of updating all the control points. This helps in keeping the computational load low in this nonlinear filter. It has also been demonstrated that SAF outperforms Volterra and FLANN filters in some nonlinear filtering tasks (Scarpiniti et al., 2015).

Most of the SAF based nonlinear filtering tasks reported in literature deals with single input single output scenarios. However, for some machine learning tasks such as classification needs multi-input multi-output nonlinear filters. A direct extension of the SAF to the multi-dimensional space leads to a complex nonlinear filtering scheme which includes a cascade of an adaptive FIR filter and an adaptive spline activation function for each sample of the feature vector. Inspired by the architecture of RBF based nonlinear filters, an attempt has been made in this paper to design a generalized spline nonlinear adaptive filter (GSNAF). In the proposed GSNAF, each element of the input signal vector is applied to an independent adaptive spline activation function directly. The outputs of the spline functions are added together to obtain a single output in the case of single output nonlinear filtering tasks. In multi-output nonlinear filtering scenarios, the multiple outputs necessary are obtained by suitably combining the spline activation function outputs. This structure removes the necessity of the adaptive FIR filter in a SAF and extends the SAF to multi-dimensional nonlinear filtering tasks.

The rest of the paper is organized as follows. The proposed generalized spline nonlinear adaptive filter is presented in Section 2. The bounds on the learning rate for the proposed scheme has also been derived in the section. A mean square analysis of the proposed nonlinear filter is made in Section 3, including a comparison between theoretical and experimental mean square error. An extensive simulation study, involving a set of classification tasks as well as dynamic system identification has been attempted in Section 4 and the concluding remarks are made in Section 5.

2. Proposed scheme

As discussed in the previous section, a SAF consists of an adaptive FIR filter in cascade with an adaptive spline function (ASF). The basic block diagram of a SAF is shown in Fig. 1. It can be observed from the figure that there are two update schemes in a SAF: one for updating the weights of the adaptive FIR filter and the second for updating the control points of the adaptive spline activation function. A spline is a usually a smooth function which passes through a set of points known as control points or knots (Schumaker, 2007). The shape of the spline function can be changed by suitably updating the control points, thereby enabling switching of SAF from a linear filter to a nonlinear filter depending on the shape of the spline activation function. The parameters for updating the control points are generated using a parameter generation unit (PGU).

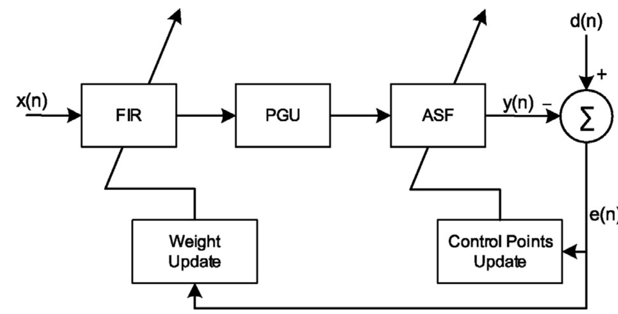

Fig. 1. Block diagram of spline adaptive filter (SAF).

A new nonlinear filter, which removes the necessity of an adaptive FIR filter in a SAF is designed in this section. The basic block diagram of a multiple input single output (MISO) configuration of the proposed GSNAF is shown in Fig. 2(a). In the figure, the input signals, which are usually elements of a feature vector in the case of a classification task, are used to generate appropriate control point update parameters using PGUs referred to as $\text{PGU}_1, \text{PGU}_2, \ldots, \text{PGU}_M$, where M is the number of inputs. The outputs of the PGUs are fed to ASFs denoted as $\text{ASF}_1, \text{ASF}_2, \ldots, \text{ASF}_M$, for the binary classification case outputs of which are combined together to obtain the filter output $y(n)$. Similarly, the block diagram of the multiple input multiple output (MIMO) GSNAF is shown in Fig. 2(b), where the outputs of ASFs are suitably combined to generate multiple filter outputs.

Consider a MISO GSNAF, where the multiple inputs are tap delayed version of a single input. Let $x(n)$ be the input signal applied to the proposed generalized spline nonlinear filter, a schematic diagram of which is shown in Fig. 3. As discussed above, the input signal is tap delayed to generate an input signal vector $\boldsymbol{x}(n) = [x(n), x(n-1), x(n-2), \cdots, x(n-j), \cdots, x(n-M+1)]^T$ of length M. Each element of the input signal vector is then passed through an adaptive spline function and the outputs of the M spline functions are added together to obtain the filter output $y(n)$. The update of the adaptive spline function requires the knowledge of the span index i and the local parameter u, which are generated using a PGU. For each element of the input signal vector, the PGU estimates the two parameters as

$$i_j = \left\lfloor \frac{x(n-j+1)}{\Delta x} \right\rfloor + \frac{Q_j - 1}{2} \tag{1}$$

and

$$u_j = \frac{x(n-j+1)}{\Delta x} - \left\lfloor \frac{x(n-j+1)}{\Delta x} \right\rfloor. \tag{2}$$

where $j = \{1, 2, \cdots, M\}$ is the index number of the input signal vector. In the above expressions, u_j and i_j are the local parameter and span index for the jth spline activation function, Q_j is the total number of control points in the jth spline function and Δx is the spacing between two control points. In this work, we have considered Δx as a non-adaptive parameter throughout the operation of the network (Scarpiniti et al., 2013; Scarpiniti, Comminiello, Scarano, Parisi & Uncini, 2016).