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Fuzzy multi-criteria acceptability analysis: A new approach to multi-criteria decision analysis under fuzzy environment

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A B S T R A C T

Uncertainty is one of the main difficulties that increases the complexity of multi-criteria decision analysis (MCDA) problems, and often uncertainty cannot be managed by probabilistic models. In such cases, the use of fuzzy methods has been successfully applied to multi-criteria decision methods in which the ranking of fuzzy quantities is crucial for the decision analysis. This paper aims to introduce a new approach to MCDA problems defined under fuzzy contexts that implements the concept of acceptability analysis, Fuzzy Multi-Criteria Acceptability Analysis (FMAA), based on the Fuzzy Rank Acceptability Analysis (FRAA), that provides a ranking and a confidence degree about the ranking of fuzzy quantities. Based on the fuzzy extension of MAVT method, the FMAA is implemented and then applied to a case study, and its results are compared with other well-known MCDA methods in order to show its validity, interpretability and consistency.

1. Introduction

Human beings face very broad and diverse decision situations, in which it is necessary to accomplish decision making processes that usually involve multiple and conflicting criteria, having led to the development of different MCDA methods (Belton & Stewart, 2002; Celik, Gul, Aydin, Gumus, & Guneri, 2015) applied to a large and growing number of real-world complex decision making problems.

Multi-criteria decision making is a complex process whose solving procedure implies different and specific difficulties that can come from various sources (Clemen, 1995) such as its inherent complexity, the uncertainty of the decision situation, the choice of suitable preference structures, setting of expert judgments within decision analysis, and so on. Therefore, the MCDA aims at supporting decision makers to make effective decisions more consistently. Despite different MCDA methods (Figueira, Greco, & Ehrgott, 2005; Izishaka & Nemery, 2013) are able to deal successfully with multi-criteria real world problems (Mardani et al., 2015), the most challenging difficulty in which such methods sometimes fail is the management of the inherent uncertainty of many of these problems. A decision analysis approach should help in identifying sources of uncertainty within the problem, representing and managing such uncertainties according to their nature. Classical decision theory provides probabilistic models to manage uncertainty in such problems, but in many of them the uncertainties have a non-probabilistic character, since they are related to imprecision and vagueness of meanings provided by experts or decision makers when they elicit their knowledge about the decision making problem (Martínez, Liu, Yang, & Herrera, 2005; Rodríguez, Labella, & Martínez, 2016). In such cases, the use of fuzzy methods, that have been included in the multi-criteria approaches (Chen & Hwang, 1992; Zamani-Sabzi, King, Gard, & Abudu, 2016), provides a useful and successful alternative (Pedrycz, Ekel, & Parreiras, 2010) for handling three main MCDA problematiques (Belton & Stewart, 2002; Figueira et al., 2005): choosing, ranking and sorting.

The management and analysis of uncertainties by fuzzy modelling in real-world decision problems implies that *“the decision taken in the fuzzy environment must be inherently fuzzy …”* (Tong & Bonissone, 1980) and not only requires the use of various fuzzy functions and models, but also inevitably leads to comparison of fuzzy quantities. Therefore, in multi-criteria decision analysis under fuzzy environments ranking of fuzzy quantities plays a key role (Bellman & Zadeh, 1970; Jain, 1976; Yuan, 1991).

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This paper aims at developing an *acceptability analysis* for fuzzy MCDA, the Fuzzy Multi-criteria Acceptability Analysis (FMAA), that is based on Fuzzy Rank Acceptability Analysis (FRAA) method, which not only results in ranking fuzzy quantities but also provides a degree of confidence about the ranking obtained that is used to determine the rank acceptability of alternatives. The FMAA will be integrated with a Fuzzy Multi-Attribute Value Theory (FMAVT) approach, that extends the ideas of Stochastic Multicriteria Acceptability Analysis (SMAA) (Lahdelma, Hokkanen, & Salminen, 1998; Lahdelma & Salminen, 2001; Tervonen & Figueira, 2008) and Probabilistic Multicriteria Acceptability Analysis (ProMAA) (Yatsalo, Gritsyuk, Mirzeabasov & Vasilevskaya, 2011; Yatsalo, Gritsyuk, Tkachuk, & Mirzeabasov, 2010b) to a fuzzy context. In such a way, FMAVT-FMAA will implement a fuzzy MCDA approach that unlike all other fuzzy MCDA approaches not only provides a ranking of alternatives but also shows how reliable is the ranking obtained by means of the degree of confidence computed by the FRAA that could result in a key tool to manage and understand the uncertainty modelled by the fuzzy information involved in the MCDA problem. Eventually, the FMAVT-FMAA will be applied to a risk management case study and its results compared with other MCDA approaches.

This paper is structured as follows: Section 2 revises briefly different fuzzy concepts, fuzzy ranking approaches, together with fuzzy relations used for comparing fuzzy numbers. Section 3 introduces the FMAVT-FMAA approach based on the fuzzy rank acceptability analysis (FRAA). Section 4 applies the FMAVT-FMAA to a case study on risk management comparing its performance with those by other MCDA methods, points out and discusses open concerns about the different proposals provided across the paper and, eventually, Section 5 concludes this paper.

2. Preliminaries

This section reviews basic notions about fuzzy sets, fuzzy preference relations and fuzzy ranking approaches, which are used in our proposed framework.

2.1. Fuzzy numbers, comparison and ranking

In this paper, the following definition of fuzzy number is considered:

Definition 1. A fuzzy number (FN) Z is a convex normal and restricted fuzzy set in $\mathbb{R}$ with a continuous or upper-continuous membership function $\mu_Z(x)$.

Therefore, we assume that there exist two real numbers $c_1, c_2 \in \mathbb{R}$, $c_1 \le c_2$, such that:

$$Z = \{(x, \mu_Z(x)) : \mu_Z(x) > 0 \quad x \in]c_1, c_2[, \;\; \mu_Z(x) = 0 \; x \notin [c_1, c_2]\}. \tag{1}$$

$\mathbb{F}$ denotes the set of all FNs, according to (1).

Remark 1. If $c = c_1 = c_2$, then $Z = c$ is a singleton and $\mu_Z(c) = 1$.

Remark 2. The condition $\mu_Z(c_1) = \mu_Z(c_2) = 0$ is, strictly speaking, not necessary and is often used for convenience, stressing the most often usage of FNs in applications.

Definition 2 (D.Dubois and Prade (1978); Wang, Ruan, and Kerre (2009))**.** Let $Z \in \mathbb{F}$ be a fuzzy number and $\alpha \in]0, 1]$. An α-cut of Z is defined as:

$$Z_\alpha = \{x \in \mathbb{R} \mid \mu_Z(x) \ge \alpha\}$$

If $\alpha = 0$ and $[A_0, B_0] = [c_1, c_2]$, then a fuzzy number Z can be identified with the family of intervals:

$$Z = \{[A_\alpha, B_\alpha]\}, (0 \le \alpha \le 1) \tag{2}$$

Let $a, b \in \mathbb{R}$ be two real numbers, a is greater than b if $a - b > 0$. For two FNs, $Z_i, Z_j \in \mathbb{F}$, a similar comparison approach based on their difference would be:

$$Z_{ij} = Z_i - Z_j$$

$$\mu_{Z_{ij}}(z) = \sup_{x,y:x-y=z} (\mu_{Z_i}(x) \wedge \mu_{Z_j}(y)) \tag{3}$$

From α-cut representation (see Eq. (2)), the proposition below is followed.

Proposition 1. *Let* $Z_i = \{[A^i_\alpha, B^i_\alpha]\}, Z_j = \{[A^j_\alpha, B^j_\alpha]\} \in \mathbb{F}$. *If* $Z_{ij} = Z_i - Z_j$ *then,*

$$Z_{ij} = \{[A^{ij}_\alpha, B^{ij}_\alpha]\} = \{[A^i_\alpha - B^j_\alpha, B^i_\alpha - A^j_\alpha]\} \tag{4}$$

The proof is straight from (D.Dubois and Prade (1978); Wang et al. (2009)).

It is shown later on that the ranking of two FNs may be based on their difference Z_{ij} by using fuzzy preference relations.

Definition 3. A fuzzy relation, R, on $\mathbb{F} \times \mathbb{F}$:

$$R = ((Z_i, Z_j), \mu_R(Z_i, Z_j)),$$

defines a membership function, $\mu_R(Z_i, Z_j) \in [0, 1]$, that provides the degree of preference of Z_i over Z_j.

An important property for fuzzy preference relations is *reciprocity* (Nakamura, 1986; Yuan, 1991), often used in ranking methods (Dubois & H., 1983; Wang & Kerre, 2001a; 2001b):

Definition 4 ((Yuan, 1991))**.** Let R be a fuzzy relation on $\mathbb{F} \times \mathbb{F}$. R is reciprocal if, for every $Z_i, Z_j \in \mathbb{F}$,

$$\mu_R(Z_i, Z_j) = 1 - \mu_R(Z_j, Z_i) \tag{5}$$

Definition 5. Let R be a fuzzy relation on $\mathbb{F} \times \mathbb{F}$. For any $Z_i, Z_j \in \mathbb{F}$, their fuzzy ranking is defined as:

$$Z_i \succeq Z_j \;\; if \;\; \mu_R(Z_i, Z_j) \ge 0.5, Z_i \succ Z_j \text{ if } \mu_R(Z_i, Z_j) > 0.5, \text{ and } Z_i \sim Z_j \text{ if } \mu_R(Z_i, Z_j) = 0.5 \tag{6}$$

For the sake of clarity, let $\mathbf{Z} = \{Z_1, \ldots, Z_n\} \subset \mathbb{F}$ be a finite family of FNs, for a fuzzy relation $R(Z_i, Z_j)$ the following notations are used:

$$\mu_{ij} = \mu_R(Z_i, Z_j) = \mu_R(Z_i \ge Z_j) = \mu_R(Z_j \le Z_i). \tag{7}$$

Remark 3. Note that the symbols $\le$, $\ge$ used here for notational purposes are different from the symbols $\preceq$, $\succeq$, which are associated with ranking of FNs.

The ranking of FNs is key in fuzzy MCDA, therefore a brief revision of different ranking approaches is provided below. A detailed review can be found in surveys (Wang & Kerre, 2001a; 2001b; Wang et al., 2009) together with their application in decision analysis, linear programming, game theory and other fields (Clemente, Fernández, & Puerto, 2011; Li, 2010; Tanaka & Asai, 1984).

The main classes of fuzzy ranking methods (apart of linguistic approaches) have been presented in Kahraman and Tolga (2009); Wang and Kerre (2001a; 2001b):

1. *Defuzzification based ranking methods*. Within these methods, FNs are represented by (defuzzified) real numbers with their subsequent ranking; well-known defuzzification methods were introduced by (Yager (1980; 1981); Adamo (1980); de Campos and González (1989)).
2. *Ranking fuzzy methods based on the distance to a reference set*. These methods define the reference sets and evaluate each fuzzy number Z_i by computing and comparing its distance to the reference set. The definition of the reference set is often