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A high-performance nonlinear dynamic scheme for the solution of equilibrium constrained optimization problems

Ahmad Golbabai*, Soraya Ezazipour

Department of Applied Mathematics, Faculty of Mathematical Sciences, Iran University of Science and Technology, P.O.Box 16846-13114, Tehran, Iran

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ABSTRACT

In this paper, a feedback neural network model is proposed to compute the solution of the mathematical programs with equilibrium constraints (MPEC). The MPEC problem is altered into an identical one-level non-smooth optimization problem, then a sequential dynamic scheme that progressively approximates the non-smooth problem is presented. Besides asymptotic stability, it is proven that the limit equilibrium point of the suggested dynamic model is a solution for the original MPEC problem. Numerical simulation of various types of MPEC problems shows the significance of the results. Moreover, the scheme is applied to compute the Stackelberg–Cournot–Nash equilibria.

1. Introduction

Mathematical programming with equilibrium constraints (MPEC) is an optimization problem in which the constraints, contain variational inequalities (VI) or complementarities. This problem emerges in numerous applications such as transportation networks, multilevel games, economic equilibrium and shape optimization (Luo, Pang, & Ralph, 1996; Marcotte, 1986; Outrata, Kocvara, & Zowe, 1998). It is well known that MPECs are very difficult problems. They are non-smooth and non-convex, even under desirable assumptions (Facchinei, Jiang, & Qi, 1999; Luo et al., 1996). Hence, standard algorithms may not be efficient for MPECs. Actually, a few numbers of numerical algorithms can solve such problems successfully (Andreani, 2001). Penalty approach, implicit programming approach and piecewise programming approach are the main methods that have been used to solve this problem (Kocvara & Outrata, 2004). However, in many engineering and scientific applications, real-time solutions are often desired, see He, Li, Huangb, and Li (2014). These problems may have high dimensions and dense structure (Ferris, 2002). Therefore, usual numerical algorithms may fail in such situations. Specially, they are not efficient in solving large-scale problems. One of the most interesting and simplest methods for solving real-time optimization is to apply neural networks.

Artificial Neural Network (ANN) is an instrument for transferring the optimization problems in a particular first order dynamic system (Malek, Hosseinipour-Mahani, & Ezazipour, 2010; Pyne, 1956). Firstly, Hopfield and Tank (1985, 1986) suggested a recurrent neural system to solve Linear programming problems. Thereafter, different models of ANNs have been utilized in dealing with various sorts of optimization problems such as convex (Malek, Ezazipour, & Hosseinipour-Mahani, 2011b; Malek et al., 2010), non-convex (Hosseinipour-Mahani & Malek, 2015; 2016; Sun & Feng, 2005), non-smooth (Hosseini & Hosseini, 2013; Liu & Wang, 2013), variational inequalities (Malek, Ezazipour, & Hosseinipour-Mahani, 2011a) and so on. The main advantage of neural computing is that these models may be implemented by a circuit easily. Furthermore, they converge to the optimal solution very quickly (Hopfield & Tank, 1985). Application of Artificial Neural networks in solving MPECs is entirely a new tackle. Recently, Sheng, Lv, and Xu (1996) considered a special class of MPEC problems, i.e., bi-level programming problem(BLP) and by using Frank-Wolfe method, they suggested an ANN model to solve this problem. A hybrid ANN for BLPs is introduced by Lana, Wena, Shihb, and Leec (2007). Based on the Morrison method Li, Li, Wu, and Huang (2014); Morrison (1968) and He et al. (2014) proposed two different models to solve convex quadratic BLP problems. Lv, Chen, and Wan (2011) have used Lagrange function to design a neural network model to solve MPEC.

The motivation behind this investigation is building up a novel ANN model with a simple structure to solve MPEC problems with general nonlinear constraints. Using KKT optimality conditions for the inner variational inequalities, MPEC problem is transformed into an identical non-smooth optimization problem. The

* Corresponding author.
E-mail addresses: golbabai@iust.ac.ir (A. Golbabai), s_ezazipour@mathdep.iust.ac.ir (S. Ezazipour).

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non-smooth constraints have been smoothed, utilizing the smoothing technique in Facchinei et al. (1999). Then, a sequential dynamic scheme which progressively approximates the non-smooth problem, is presented. The use of Lagrange multipliers in Lagrange ANN model in Lv et al. (2011), increase the number of state variables dramatically, which enlarge the scale of ANN model. Thus, reducing the scale of neural network for solving MPECs, is very necessary. The outstanding feature of the proposed model is not included Lagrange multipliers and hence have a simpler framework for implementing. Moreover, it is more suitable for large-scale problems.

This research paper is structured as follows. In the next section, The problem formulation and smoothing method are explained. In Section 3, first, the energy function and the corresponding gradient-based subnetwork is constructed . Next, the theoretical aspects of suggested model are discussed. A feedback ANN model for solving MPEC problems is presented in Section 4. Section 5, is allocated to computational experiences on several academic examples and a Stackelberg–Cournot–Nash problem. Section 6, concludes this paper.

2. Problem statement and smoothing technique

Let $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $f : \mathbb{R}^{n+m} \to \mathbb{R}$, $g : \mathbb{R}^n \to \mathbb{R}^p$ are continuously differentiable functions. Also, for each $x \in Z = \{x \in \mathbb{R}^n : g(x) \le 0\} S(x)$ denotes the solutions of variational inequality problems defined by a continuously differentiable function $F(x, y)$ over the set $\Gamma(x)$ represented by

$$(u - y)^T F(x, y) \ge 0, \qquad \forall u \in \Gamma(x),$$

where $\Gamma(x)$ is defined by $\Gamma(x) = \{y \in \mathbb{R}^m : h_j(x, y) \ge 0, j = 1, \dots, l\}$, with $h : \mathbb{R}^{n+m} \to \mathbb{R}^l$ twice continuously differentiable and concave over y. The general formulation of the mathematical program with equilibrium constraints (MPEC) can be written as:

$$\begin{array}{ll} \min & f(x, y) \\ \text{s.t.} & g(x) \le 0, \\ & y \in S(x). \end{array} \qquad (1)$$

Let $J(x, y) = \{j : h_j(x, y) = 0\}$ be the set of active constraints and make the following assumptions (see also Facchinei et al., 1999):

A1. $Z \subseteq \mathbb{R}^n$ is nonempty and compact. $\Gamma(x) \neq \emptyset \quad \forall x \in A$, where A is an open set containing Z.
A2. $\Gamma(x)$ is uniformly compact on A, i.e., there exist an open bounded set $B \subseteq \mathbb{R}^m$ such that $\Gamma(x) \subseteq B \quad \forall x \in A$.
A3. F is uniformly strongly monotone over the second variable on $A \times B$, i.e., there exist a constant $\alpha \ge 0$ such that

$$d^T \nabla_y F(x, y) d \ge \alpha \|d\|^2, \quad \forall\ (x, y) \in A \times B \quad \text{and} \quad d \in \mathbb{R}^m.$$

A4. At each $x \in Z$ and $y \in S(x)$, the partial gradients $\nabla_y h_j(x, y), \quad j \in J(x, y)$ are linearly independent.

The prominent consequence of the above assumptions is that MPEC Problem (1) can be reformulated as an equivalent one-level non-smooth and non-convex optimization problem (Facchinei et al., 1999), i.e.

$$\begin{array}{ll} \min & f(x, y) \\ \text{s.t.} & g(x) \le 0, \\ & F(x, y) - \nabla_y h(x, y)\lambda = 0, \\ & h(x, y) \ge 0, \quad \lambda \ge 0, \quad \lambda^T h(x, y) = 0. \end{array} \qquad (2)$$

It is easy to see that, the complementarity-type constraints of the Problem (2) are unable to satisfy a standard constraint qualification which is necessary for the regularity of a nonlinear optimization problem (Andreani, 2001; Luo et al., 1996). This causes great difficulty to use the ANN models for solving Problem (2).

To deal with this difficulty, this non-convex optimization problem is transferred into a non-smooth equivalent reformulation of the Problem (2) as follows:

$$\begin{array}{ll} \min & f(x, y) \\ \text{s.t.} & g(x) \le 0, \\ & F(x, y) - \nabla_y h(x, y)\lambda = 0, \\ & h(x, y) - z = 0, \\ & -2\min(z, \lambda) = 0. \end{array} \qquad (3)$$

Where $z \in \mathbb{R}^l$ and $\min(z, \lambda) = (\min(z_1, \lambda_1), \dots, \min(z_l, \lambda_l))$. The necessity of the multiplicative factor -2 before the minimum function will shortly appear. Following the smoothing method presented in Facchinei et al. (1999), suggested ANN model for solving MPEC problems is presented in the next sections.

Let $\epsilon \ge 0$ is a scalar. The function $\phi : \mathbb{R}^2 \to \mathbb{R}$, defined by

$$\phi_\epsilon(u, v) = \sqrt{(u - v)^2 + 4\epsilon^2} - (u + v),$$

has following significant properties:

(i) $\phi_\epsilon(u, v) = 0 \iff u \ge 0, \quad v \ge 0, \quad uv = \epsilon^2$.
(ii) For every $\epsilon \neq 0$, the function $\phi_\epsilon(u, v)$ is smooth for every u, v.
(iii) $\lim_{\epsilon \to 0} \phi_\epsilon(u, v) = -2min(u, v), \ \forall\ (u, v) \in \mathbb{R}^2$. Hence, $\phi_\epsilon(u, v)$ is a smooth perturbation of the minimum function.

Define the following nonlinear functions $H : \mathbb{R}^{n+m+2l} \to \mathbb{R}^p$ and $G_\epsilon : \mathbb{R}^{n+m+2l} \to \mathbb{R}^{m+2l}$,

$$H(w) = H(x, y, z, \lambda) = h(x),$$

$$G_\epsilon(w) = G_\epsilon(x, y, z, \lambda) = \begin{pmatrix} F(x, y) - \nabla_y h(x, y)\lambda \\ h(x, y) - z \\ \phi_\epsilon(z, \lambda) \end{pmatrix},$$

where

$$\phi_\epsilon(z, \lambda) = (\phi_\epsilon(z_1, \lambda_1), \dots, \phi_\epsilon(z_l, \lambda_l))^T.$$

It can be easily seen that for every $\epsilon \neq 0$, G_ϵ is locally Lipschitz continuous and regular. Then, the Problem (3) can be approximated by

$$\begin{array}{ll} \min & f(w) \\ \text{s.t.} & H(w) \le 0, \\ & G_\epsilon(w) = 0. \end{array} \qquad (4)$$

Problem (4) can be considered as a perturbation of Problem (3) which is a smooth optimization problem. Thus, we dominate the difficulty that Problem (3) does not satisfy any regularity assumptions.

Definition 1. (Lv et al., 2011) Let w be a feasible point of Problem (4) and $L = \{j : H_j(w) = 0, \ j = 1, \dots, l\}$. We say that w is a regular point if the gradients $\nabla G_{\epsilon_1}(w), \dots, \nabla G_{\epsilon_{m+2l}}(w)$ and $\nabla H_j(w), \ j \in L$ are linearly independent.

The relationship between the Problem (4) and Problem (1) is clarified in the next theorem.

Theorem 1. *(Lv et al., 2011) Let $\{w^\epsilon\}$ be a sequence of solutions of Problem (4). Suppose that $\{w^\epsilon\}$ converges to w^* for $\epsilon \to 0^+$. if w^* is a regular point, then w^* is a strong C-stationary point of the MPEC problem (1).*

3. Energy function and neural subnetwork

Regarding formulating MPEC problem in terms of a neural network, the basic procedure is to design a proper energy function such that the lowest energy state corresponds to the optimal solution. Based on this function, a gradient system of first order differential equations which corresponds to our ANN model, is considered.