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Evidence combination rule with contrary support in the evidential reasoning approach

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ABSTRACT

Three aspects of problems such as reasonable weight constraint, cumulative weight effect and relative weight equivalence cannot be well reflected in the evidential reasoning (ER) approach. In order to solve the above problems, a contrary support is defined to restrict the degree influenced by the evidence to be combined in combination process. Then pair-weighted and cumulative pair-weighted discounting methods are presented to generate basic probability assignment (BPA) for evidence. Pair-wised and recursive combination rules are established to make combination with the BPA of evidence by orthogonal sum operation and several theorems such as relative weight equivalence are proved. A combination algorithm is proposed to solve multiple criteria decision making (MCDM) problems by integrating pignistic probability and expected utility with the established combination rules. Finally, an illustrative example is provided to demonstrate the applicability of the proposed combination rules and algorithm.

1. Introduction

The Evidential Reasoning (ER) approach is developed on the basis of Dempster–Shafer theory (DST) of evidence (Shafer, 1996) and decision theory, and it is generally used to analyze multiple criteria decision-making (MCDM) problems under uncertainties (Yang & Singh, 1994; Yang & Xu, 2002a). In the published literature, we found two versions of the ER approach. The earlier version utilizes the original concept of evidential reasoning for criteria aggregation with the intent of discovering the link between MCDM and DST (Pratyush & Yang, 1994; Yang & Singh, 1994). However, the reasoning process in this version of the ER approach is only approximate, which causes the generated outcomes to assume limited compensation among criteria, and is inappropriate in decision-making environments where more complete compensations among criteria are required and multiple pieces of evidence are often in conflict with each other. In order to overcome the above drawbacks, the ER approach is further modified mainly from two aspects. Firstly, the appropriate compensations (among criteria) are well considered in the reasoning process by establishing a new evidence combination rule. Secondly, different types of data can be handled by a set of utility-based information transformation techniques (Yang & Xu, 2002b). This version of ER approach is capable of handling both qualitative and quantitative information, probabilistic uncertainty, incomplete information, and complete ignorance in some assessments, and is considered as an advanced ER approach. The succedent research uses the advanced ER approach to handle various types of problems, such as uncertainties that are interval or fuzzy in nature (Xu, Yang, & Wang, 2006), the co-existence of uncertainties in various parameters (Xu et al., 2006; Zhang, Wang, Li, & Chen, 2017), fuzzy linguistic assessment grades (Yang, Wang, Xu, & Chin, 2006), interval belief degrees (Wang, Yang, Xu, & Chin, 2006; 2007), interval uncertainties co-existence in both weights of criteria and belief degrees (Fu & Chin, 2014; Fu & Wang, 2015; Guo, Yang, Chin, & Wang, 2007), belief degrees assigned to fuzzy linguistic assessment grades and interval assessment grades (Chen, Cheng, & Chiou, 2016; Chin & Fu, 2014; Guo, Yang, Chin, Wang, & Liu, 2009), and etc. Furthermore, the ER approach and its extensions have been widely applied to solve some practical problems in different fields such as medical quality assessment (Kong, Xu, Yang, & Ma, 2015), technical analysis in forex trading expert system (Dymova, Sevastjanov, & Kaczmarek, 2016), trauma outcome prediction (Kong et al., 2016), smart home subcontractor selection (Polat, Cetindere, Damci, & Bingol, 2016), environmentally-friendly designs selection (NG, 2016), navigational risk assessment (Zhang, Yan, Zhang, Yang, & Wang, 2016), energy system optimization (Zhang et al., 2016), data classification (Xu, Zheng, Yang, Xu, & Chen, 2017), and etc.

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It is important to note that the core of each version of the ER approach is the evidence combination rule, which is rooted in probability theory and constitutes a conjunctive probabilistic inference process. The earlier version of the ER approach employs the Dempster's rule for criteria aggregation with the introduction of criteria weights in the probability mass assignment, while the later version establishes a new evidence combination rule by revising Dempster's rule with an innovative weight normalization process. Each version of the combination rule employs orthogonal sum as a basis for evidence aggregation. As a result, both versions generalize Bayes' rule and follow the DST framework. Compared with the earlier combination rule (Dempster's rule), one of the greatest contribution of the later combination rule in the advanced ER approach (ER's rule) lies in its recognition of the difference between the residual support generalized by Shafer's discounting method and the degree of global ignorance denoted by frame of discernment. In the ER's rule, the global ignorance in a piece of evidence is considered as an intrinsic property and has no relevance to other evidence, while the residual support is considered as an extrinsic feature of the evidence and is incurred due to the relative importance of the evidence compared with other evidence (Yang & Xu, 2013). Consequently, if the residual support and global ignorance are confused in the basic probability assignment (BPA) calculation or the process of combining evidence, the results will be unreasonable.

The DST is used to combine two pieces of non-compensatory evidence so that if either of them completely opposes a proposition, the proposition will not be supported at all, no matter how strongly it may be supported by other evidence (Mondéjar-Guerra, Muñoz-Salinas, Marín-Jiménez, Carmona-Poyato, & Medina-Carnicer, 2015). The ER approach differs from the DST in that it is used to solve MCDM problems, in which the evidence generated from each criterion is compensatory. It attempts to reflect the compensation among criteria by the ER's discounting and recursive procedure, but the characteristics of MCDM problems such as reasonable weight constraint, cumulative weight effect and relative weight equivalence are not well considered and reflected. The reasonable weight constraint means that the weight of a criterion or a piece of evidence is the important degree for a special fusion problem to be solved, so as to the role played by the combined evidence should be equal to its weight in the combination process. The cumulative weight effect means that the importance of the combination result for several pieces of evidence should be proportional to the sum of their criteria weights. The relative weight equivalence refers to once the relative importance degrees of criteria are defined (e.g., $w_1 : w_2 = 4 : 1$), the normalized evaluation results of an alternative under different groups of weight values with the same relative importance degrees (e.g., $\{w_1 = 1, w_2 = 0.25\}$ and $\{w_1 = 0.8, w_2 = 0.2\}$) should be equivalent. The above three aspects of characteristics are often involved in lots of MCDM methods (Mondéjar-Guerra et al., 2015; Saaty, 2013), but they are not considered in the ER approach at all, and there have been no attempts to improve the ER approach from the above three perspectives until now.

In this paper, the ER approach will be further developed to take into account the above three aspects of characteristics in MCDM problems, resulting in a new evidence combination rule with contrary support. In particular, a pair-wised contrary support-based discounting method is proposed to hold reasonable weight constraint, a recursive combination rule used to make combination is proposed to hold cumulative weight effect and relative weight equivalence, and finally a combination algorithm is established to solve MCDM problems by integrating the pignistic probability and expected utility with the established combination rules. The rest of this paper is organized as follows. Section 2 introduces the preliminary details of the DST and the ER approach. Section 3 defines the concept of contrary support, explains the evidence combination rules with contrary support, and establishes a combination algorithm. Section 4 provides an illustrative example to demonstrate the detailed implementation process of the proposed rules and algorithm. Finally, the paper is concluded in Section 5.

2. Preliminaries

This work is established on the basis of the DST and the advanced ER approach, thus it is necessary to provide a brief introduction to the prior knowledge used as the foundation for later discussions. The DST was first proposed by Dempster in the 1960s and mathematically developed by his student Shafer in the 1970s (Dempster, 1967; Shafer, 1996). It provides a distributed framework to model probabilistic uncertainties, based on several key concepts such as frame of discernment, basic probability assignment function, and etc.

Definition 1 (Wang, Yang, Xu, & Chin, 2007)**.** Suppose a possible hypothesis of variable is $\theta_n (n = 1, \ldots, N)$, each of possible hypotheses is exclusive, then a finite nonempty exhaustive set of all possible hypotheses $\Theta = \{\theta_1, \ldots, \theta_N\}$ is called frame of discernment, and its power set that consists of 2^N subsets of Θ is usually expressed as

$$P(\Theta) = 2^\Theta = \{\varnothing, \theta_1, \ldots, \theta_N, \{\theta_1, \theta_2\}, \ldots, \{\theta_1, \theta_N\}, \ldots, \{\theta_1, \ldots, \theta_{N-1}\}, \Theta\} \tag{1}$$

Definition 2 (Xu et al., 2006)**.** Suppose $\Theta = \{\theta_1, \ldots, \theta_N\}$ is the frame of discernment, if the mapping function m: $2^\Theta \to [0, 1]$ could fulfill

$$\begin{cases} m(\theta) = 0 & \theta = \varnothing \\ m(\theta) \geq 0, \sum\limits_{\theta \subseteq \Theta} m(\theta) = 1 & \theta \neq \varnothing \end{cases} \tag{2}$$

then $m(\cdot)$ is called basic probability assignment (BPA) function of Θ. If $m(\theta) > 0$, θ is named as a focal element.

The belief assigned to the empty set is defined as zero in the BPA definition as in Eq. (2), and this is accepted and holds in most of combination rules, so $m(\varnothing) = 0$ will be omitted and not described in equations in the following contents unless specially stated. Based on the arguments in Shafer's work, the BPA function has already taken into account the weight of evidence and it is an aggregation of the belief distribution and the weight of evidence. The weight of evidence is usually denoted by w_i ($0 \leq w_i \leq 1$) with $w_i = 0$ and 1 denoting "not important at all" and "the most important" respectively. When the BPA functions of several pieces of evidence are obtained, they are capable of combining by Dempster's rule.

Definition 3 (Yang & Xu, 2013)**.** Suppose $(\theta, p_{\theta,i})$ shows that the evidence e_i points to proposition θ to a belief degree $p_{\theta,i}$, then the profiled expression

$$b_i = \left\{ \left(\theta, p_{\theta,i}\right), \forall\theta \subseteq \Theta, \sum_{\theta \subseteq \Theta} p_{\theta,i} = 1 \right\} \tag{3}$$

is called the belief distribution (BD) of e_i.

Definition 4 (Shafer, 1996)**.** Suppose the BD of evidence e_i is b_i as in Eq. (3), w_i is the weight of the evidence e_i used to discount b_i, then the Shafer's discounting method can be defined to generate BPA for the evidence e_i as follows:

$$m_i(\theta) = \begin{cases} w_i p_i(\theta) & \theta \subset \Theta \\ w_i p_i(\theta) + (1 - w_i) & \theta = \Theta \end{cases} \tag{4}$$

Definition 5 (Dempster, 1967)**.** Suppose the BPA functions of two pieces of evidence are m_1 and m_2 on Θ, $\oplus$ is the orthogonal sum